Karlsruher Institut für Technologie

Hochperformante Humanoide Technologien

Robotics I: Introduction to Robotics
Exercise 5 – Grasping

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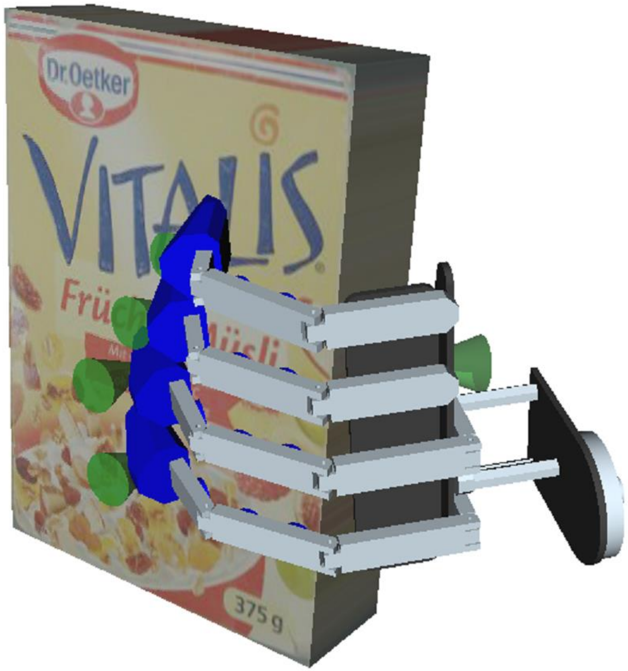
http://www.humanoids.kit.edu

Grasping - Exercises

- 1. Friction Triangles
- 2. Grasp Wrench Space
- 3. Force Closure
- 4. Medial Axes

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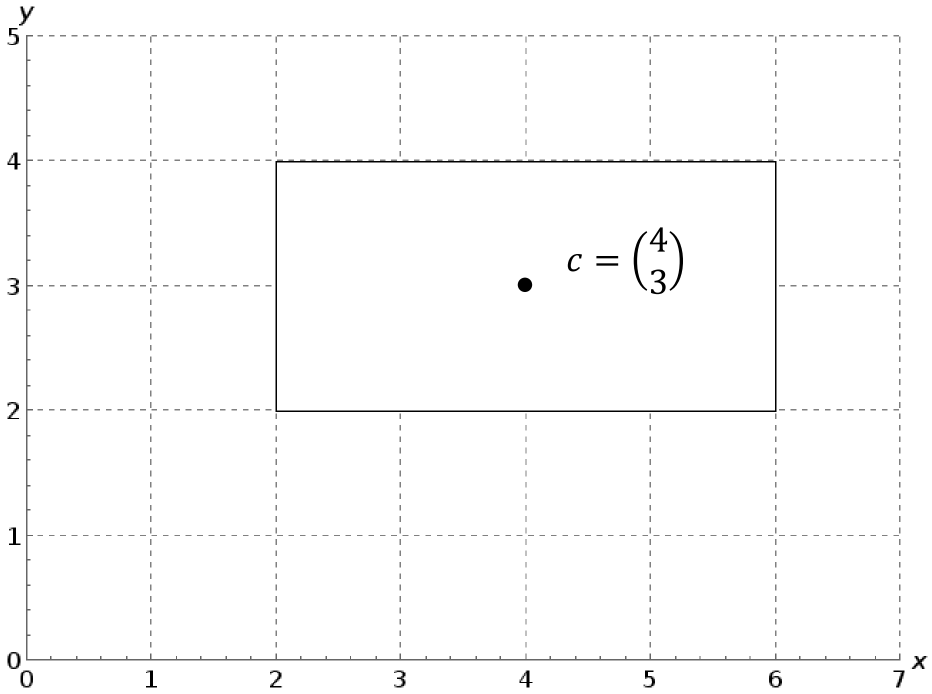
Exercise 1: Friction Triangles

- Two-dimensional object with center of mass c
- Point contacts with friction
- Contact forces are represented by friction triangles

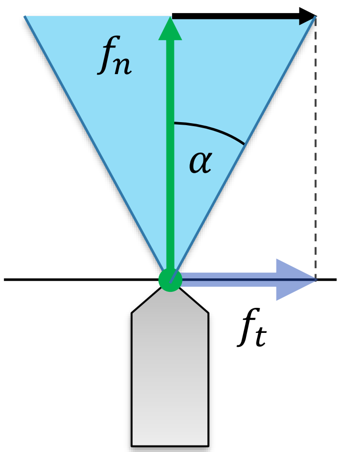
1. Opening angle α for a friction triangle with $\mu = 1$
2. Draw normal forces and corresponding friction triangles
3. Determine force vectors at the edges of the friction triangles



Exercise 1.1: Opening Angle of a Friction Triangle

- Determine the opening angle α of a friction triangle assuming a friction coefficient $\mu = 1$.

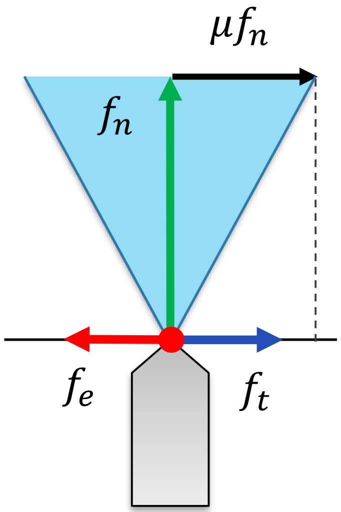
$\alpha =$



Contact Models: Coulomb‘s Law of Friction

- Empirical Law
- Describes the relation of the tangential force f_t to the normal force f_n:

$$f_t \leq \mu \cdot f_n$$



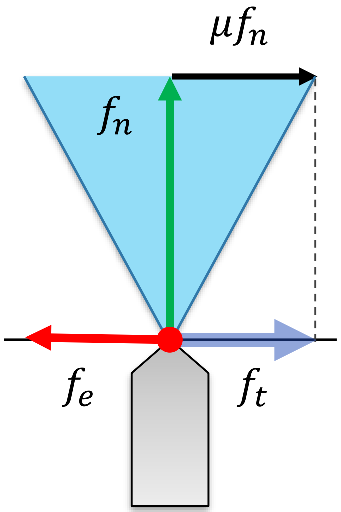
Friction coefficient $\mu > 0$ (material dependent)

- For static contact:
  - $f_t < \mu \cdot f_n$
  - Tangential force acts against an applied force f_e.

Contact Models: Coulomb‘s Law of Friction

- Empirical Law
- Describes the relation of the tangential force f_t to the normal force f_n:

$$f_t \leq \mu \cdot f_n$$

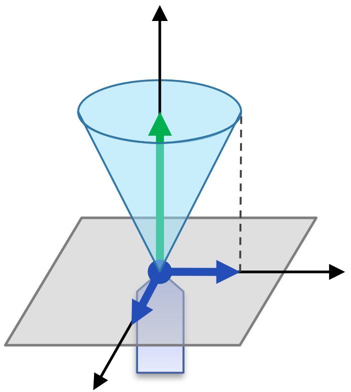


- **Friction coefficient $\mu > 0$** (material dependent)
- A contact starts sliding if:
  - $f_e > f_t = \mu \cdot f_n$
  - Tangential force acts against direction of motion.
  - **Note:** The friction coefficient for sliding friction may differ from the friction coefficient for static friction!

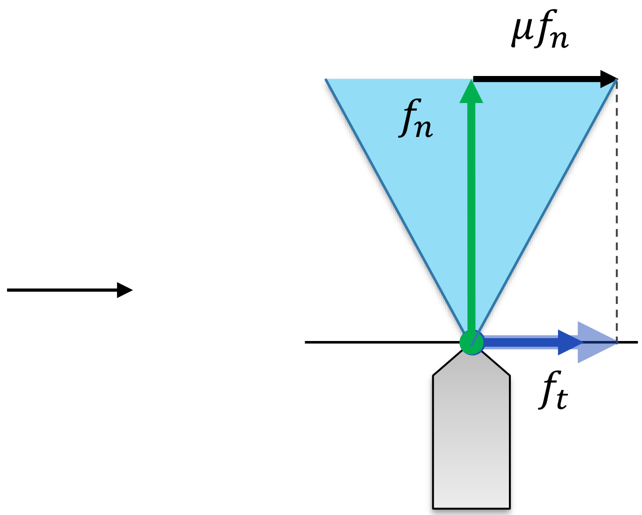
Contact Models: Coulomb‘s Law of Friction

- Empirical Law
- Describes the relation of the tangential force f_t to the normal force f_n:

Rigid contact with friction



3D: Friction Cone



2D: Friction Triangle

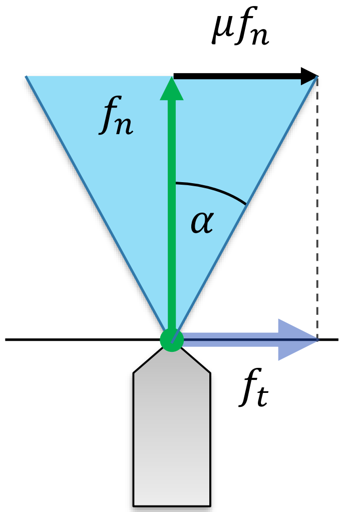
Exercise 1.1: Opening Angle of a Friction Triangle

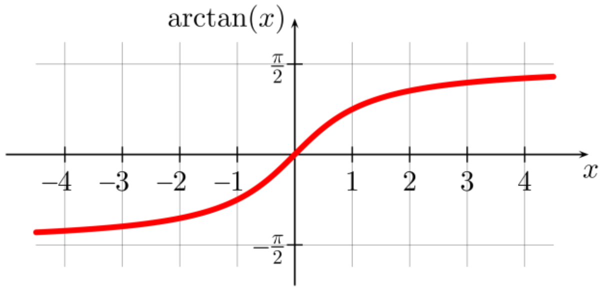
- Determine the opening angle α of a friction triangle assuming a friction coefficient $\mu = 1$.

$$\tan(\alpha) = \frac{\mu f_n}{f_n} = \mu$$

$$\alpha = \arctan(\mu)$$

$$= \arctan(1) = \frac{\pi}{4} = 45°$$





Exercise 1.1: Bonus

- How could we determine the friction coefficient between two materials?

$$f_n = \cos(\alpha) \cdot f_g$$

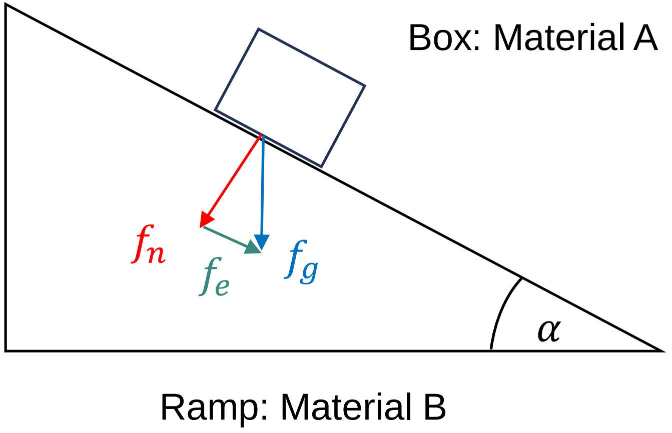
$$f_e = \sin(\alpha) \cdot f_g$$

The box starts sliding if

$$f_e = \mu \cdot f_n$$

$$\sin(\alpha) = \mu \cdot \cos(\alpha)$$

$$\tan(\alpha) = \mu$$



Exercise 1.2: Drawing Friction Triangles

- Contact points:

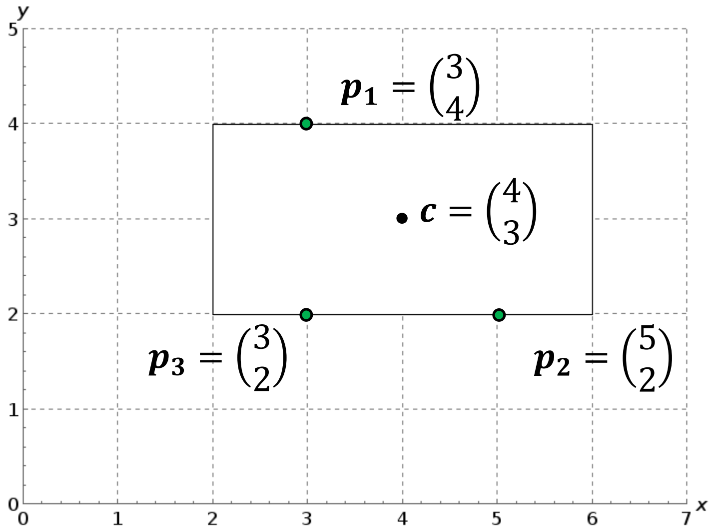
$$\boldsymbol{p}_1 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \quad \boldsymbol{p}_2 = \begin{pmatrix} 5 \\ 2 \end{pmatrix}, \quad \boldsymbol{p}_3 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

- Corresponding force vectors:

$$\boldsymbol{f}_1 = \begin{pmatrix} 0 \\ -0.5 \end{pmatrix}, \boldsymbol{f}_2 = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}, \boldsymbol{f}_3 = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$$

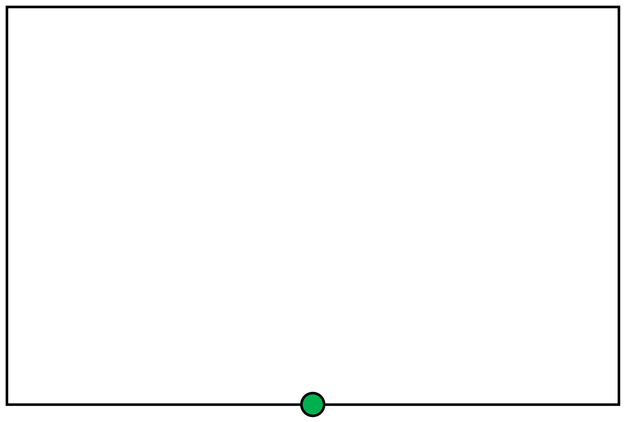
- Task: Draw the force vectors and the corresponding friction triangles.

$$\alpha = \arctan(\mu) = 45°$$



Exercise 1.2: Drawing Friction Triangles

- Draw normal force



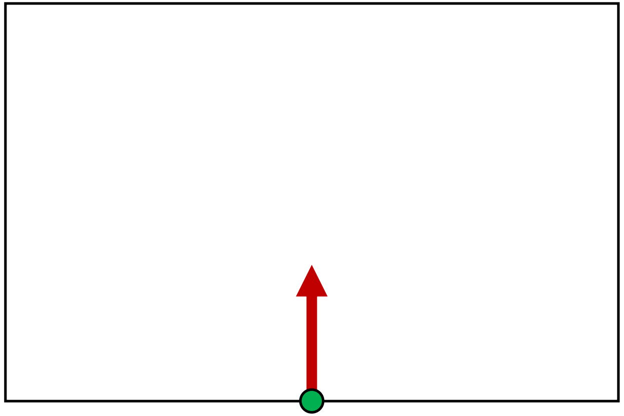
$$f = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$$

Exercise 1.2: Drawing Friction Triangles

- Draw normal force

- Opening angle:

$$\alpha = \arctan(\mu)$$



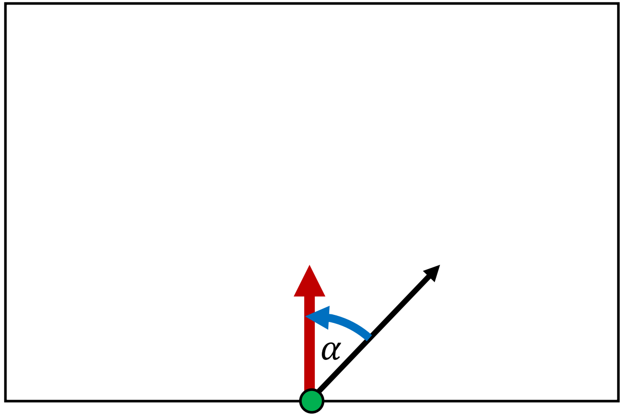
$$f = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$$

Exercise 1.2: Drawing Friction Triangles

- Draw normal force
- Opening angle:

$$\alpha = \arctan(\mu)$$

- Draw triangle



$$f = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$$

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- Draw normal force

- Opening angle:

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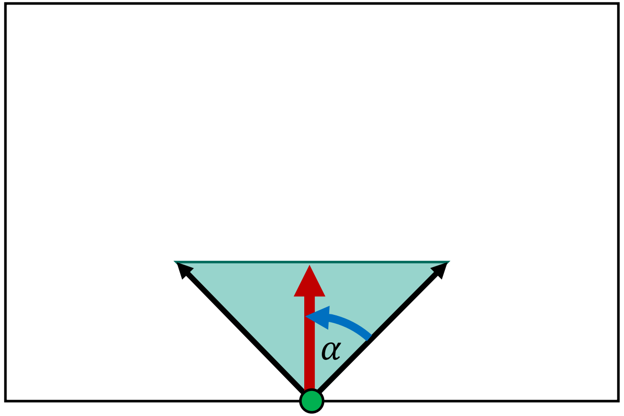
- Draw triangle



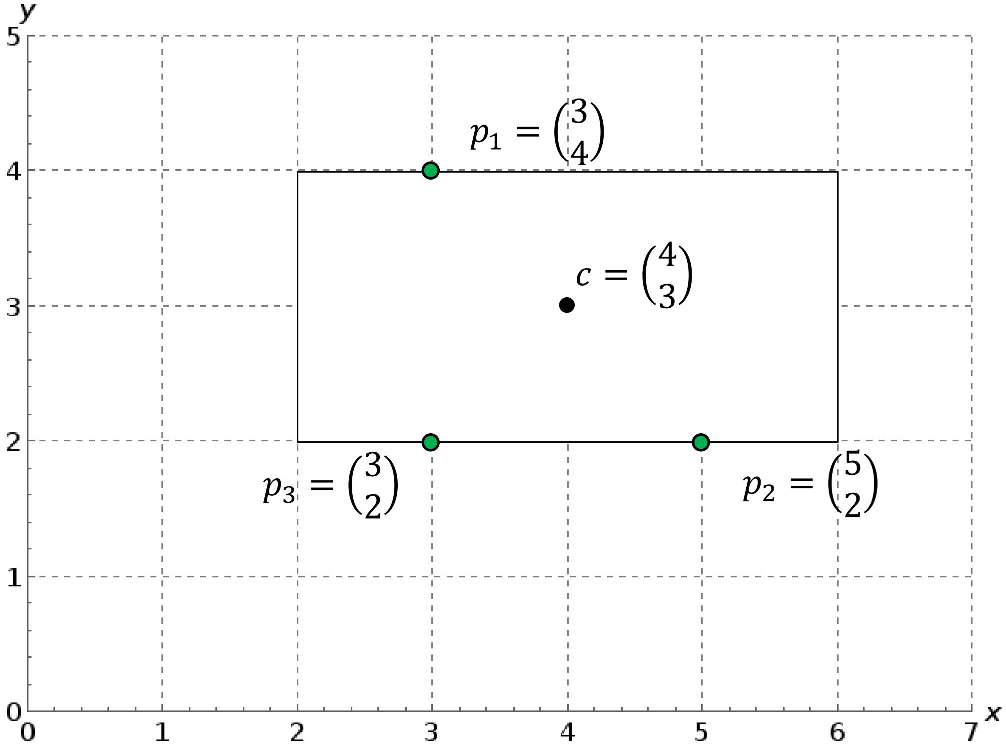
$$f = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$$

Exercise 1.2: Drawing Friction Triangles

- Force vectors

$$f_1 = \begin{pmatrix} 0 \\ -0.5 \end{pmatrix},$$
$$f_2 = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix},$$
$$f_3 = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$$

- $\alpha = 45°$



Exercise 1.2: Drawing Friction Triangles

- Force vectors

$$f_1 = \begin{pmatrix} 0 \\ -0.5 \end{pmatrix},$$
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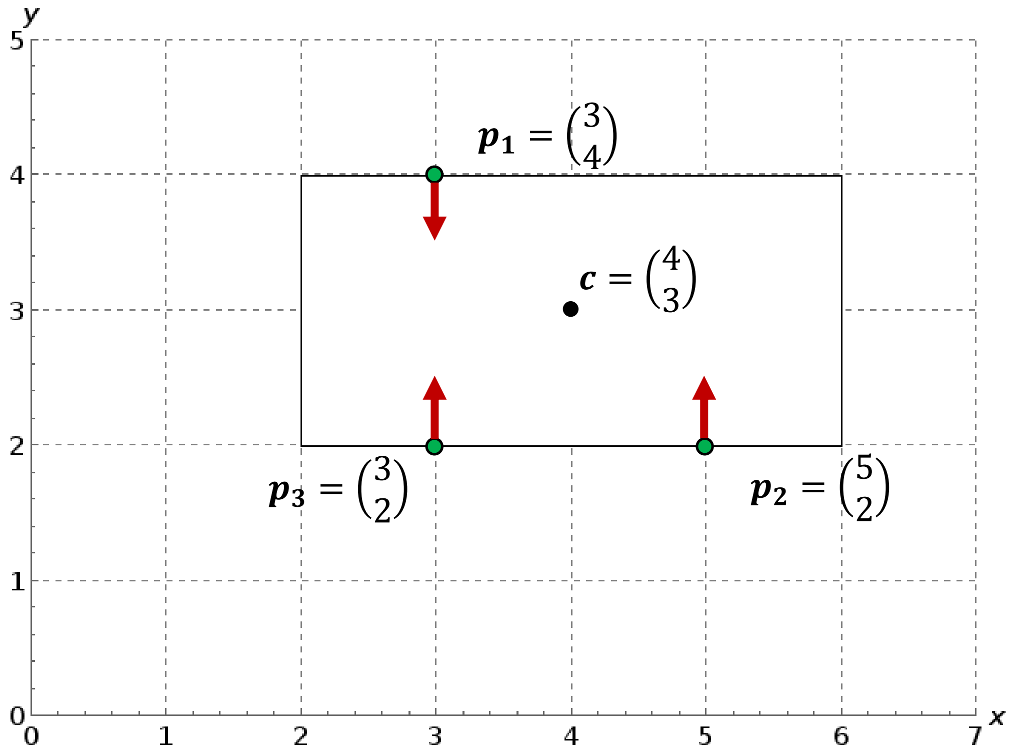

Exercise 1.2: Drawing Friction Triangles

- Force vectors

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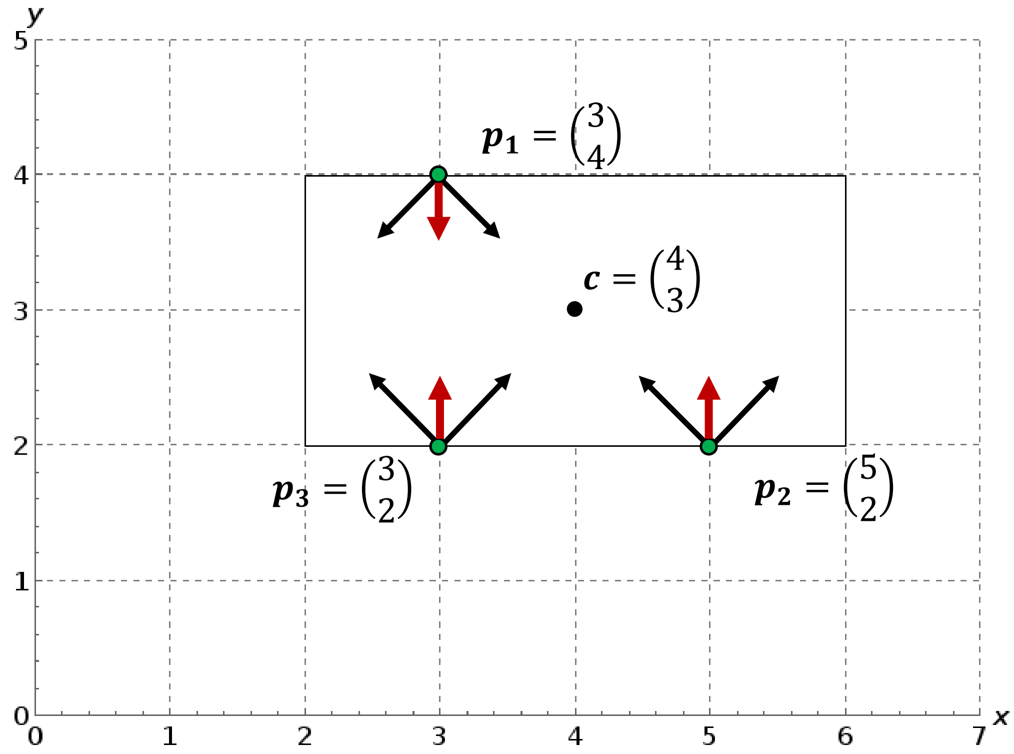

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- Force vectors

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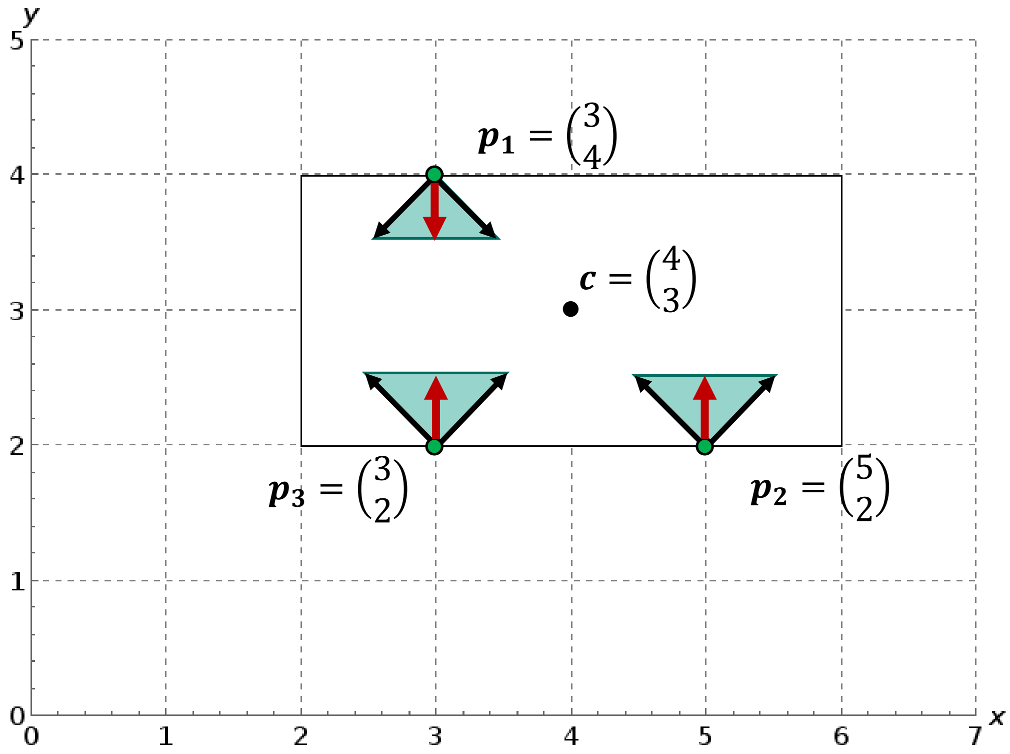
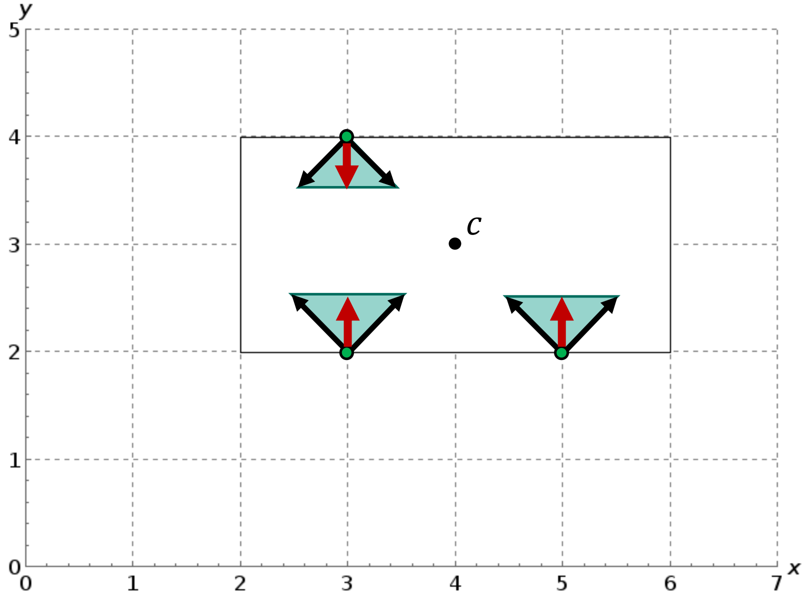
- $\alpha = 45°$



Exercise 1.3: Force Vectors at the Edges

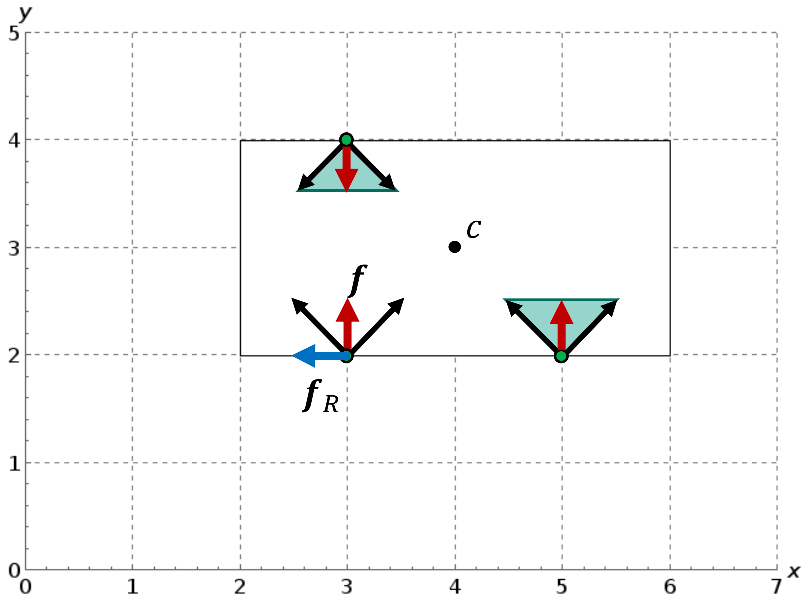
- Determine the force vectors at the edges of the friction triangles



Exercise 1.3: Force Vectors at the Edges

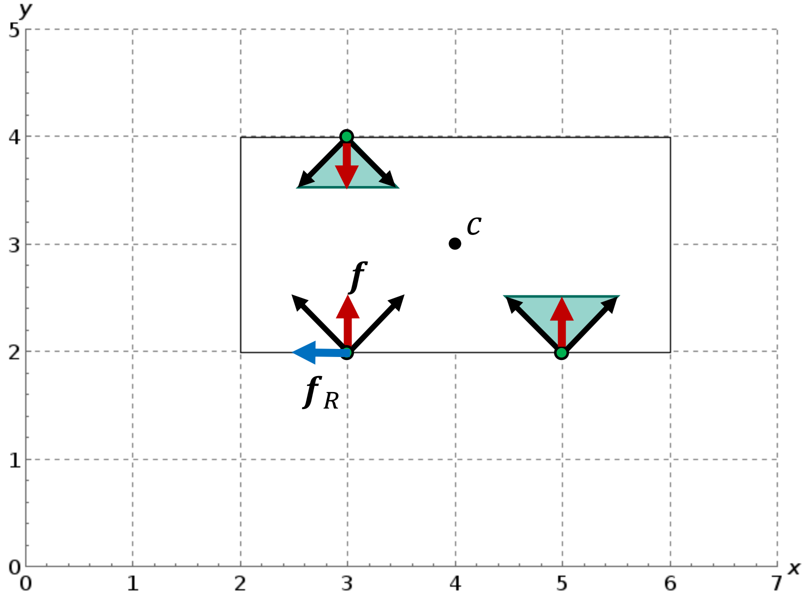
- Determine the force vectors at the edges of the friction triangles.

- Force of friction $\boldsymbol{f}_R$ acts perpendicular to $\boldsymbol{f}$



Exercise 1.3: Force Vectors at the Edges

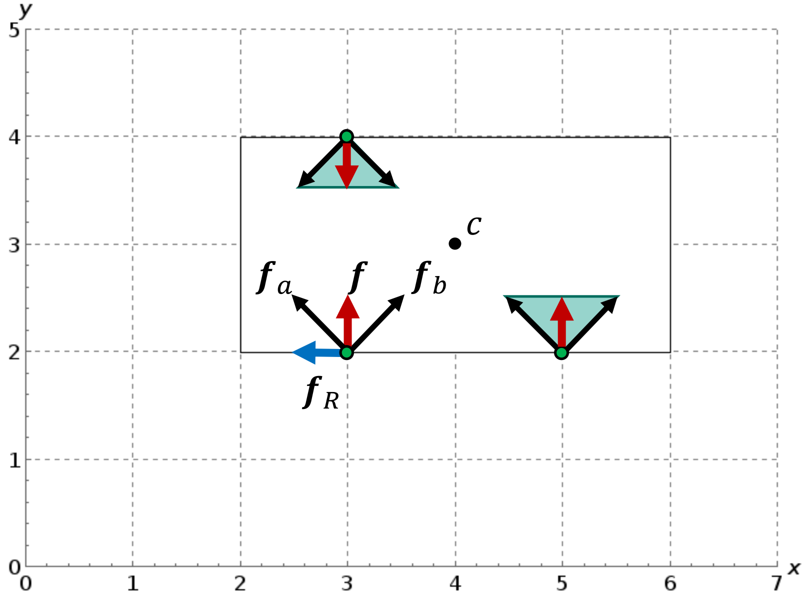
- Determine the force vectors at the edges of the friction triangles.
- Force of friction $\boldsymbol{f}_R$ acts perpendicular to $\boldsymbol{f}$
- $||\boldsymbol{f}_R|| = \mu \cdot ||\boldsymbol{f}\,||$



Exercise 1.3: Force Vectors at the Edges

- Determine the force vectors at the edges of the friction triangles.
- Force of friction $\boldsymbol{f}_R$ acts perpendicular to $\boldsymbol{f}$
- $||\boldsymbol{f}_R|| = \mu \cdot ||\boldsymbol{f}\,||$
- Force vectors at the edges:

$$\boldsymbol{f}_a = \boldsymbol{f} + \boldsymbol{f}_R$$
$$\boldsymbol{f}_b = \boldsymbol{f} - \boldsymbol{f}_R$$

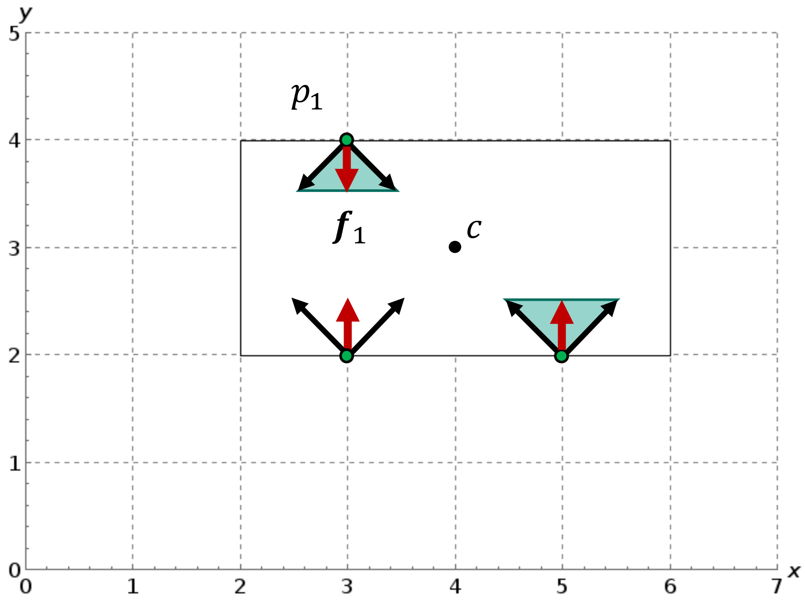


Exercise 1.3: Force Vectors at the Edges

$\boldsymbol{f}_R \perp \boldsymbol{f},\ \|\boldsymbol{f}_R\| = \mu \cdot \|\boldsymbol{f}\|,\ \mu = 1$

$\boldsymbol{f}_a = \boldsymbol{f} + \boldsymbol{f}_R,\ \boldsymbol{f}_b = \boldsymbol{f} - \boldsymbol{f}_R$

$\boldsymbol{f}_1 = \begin{pmatrix} 0 \\ -0.5 \end{pmatrix}$



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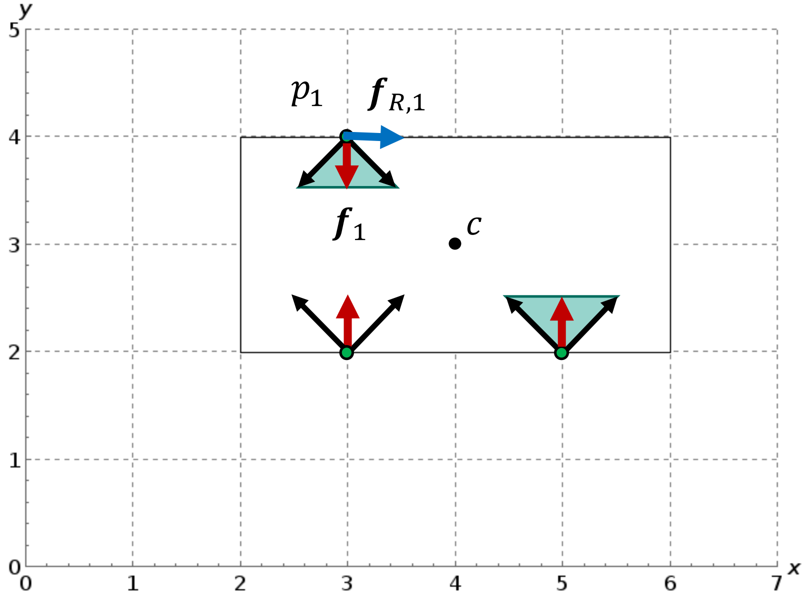
$\boldsymbol{f}_R \perp \boldsymbol{f}, \|\boldsymbol{f}_R\| = \mu \cdot \|\boldsymbol{f}\|, \mu = 1$

$\boldsymbol{f}_a = \boldsymbol{f} + \boldsymbol{f}_R, \boldsymbol{f}_b = \boldsymbol{f} - \boldsymbol{f}_R$

$$\boldsymbol{f}_1 = \begin{pmatrix} 0 \\ -0.5 \end{pmatrix}$$

$$\boldsymbol{f}_{\perp,1} = \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}$$



Exercise 1.3: Force Vectors at the Edges

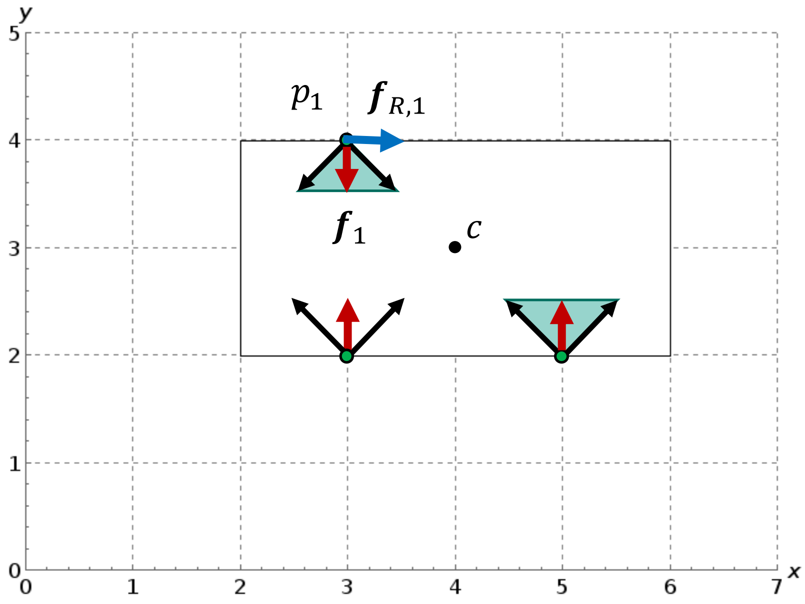
$\boldsymbol{f}_R \perp \boldsymbol{f}$, $\|\boldsymbol{f}_R\| = \mu \cdot \|\boldsymbol{f}\|$, $\mu = 1$

$\boldsymbol{f}_a = \boldsymbol{f} + \boldsymbol{f}_R$, $\boldsymbol{f}_b = \boldsymbol{f} - \boldsymbol{f}_R$

$$\boldsymbol{f}_1 = \begin{pmatrix} 0 \\ -0.5 \end{pmatrix}$$

$$\boldsymbol{f}_{\perp,1} = \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}$$

$$\boldsymbol{f}_{R,1} = \mu \cdot \boldsymbol{f}_{\perp,1} = 1 \cdot \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}$$



Exercise 1.3: Force Vectors at the Edges

$$\boldsymbol{f}_R \perp \boldsymbol{f},\ \|\boldsymbol{f}_R\| = \mu \cdot \|\boldsymbol{f}\|,\ \mu = 1$$

$$\boldsymbol{f}_a = \boldsymbol{f} + \boldsymbol{f}_R,\ \boldsymbol{f}_b = \boldsymbol{f} - \boldsymbol{f}_R$$

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$$\boldsymbol{f}_{\perp,1} = \begin{pmatrix} 0.5 \\ 0 \end{pmatrix}$$

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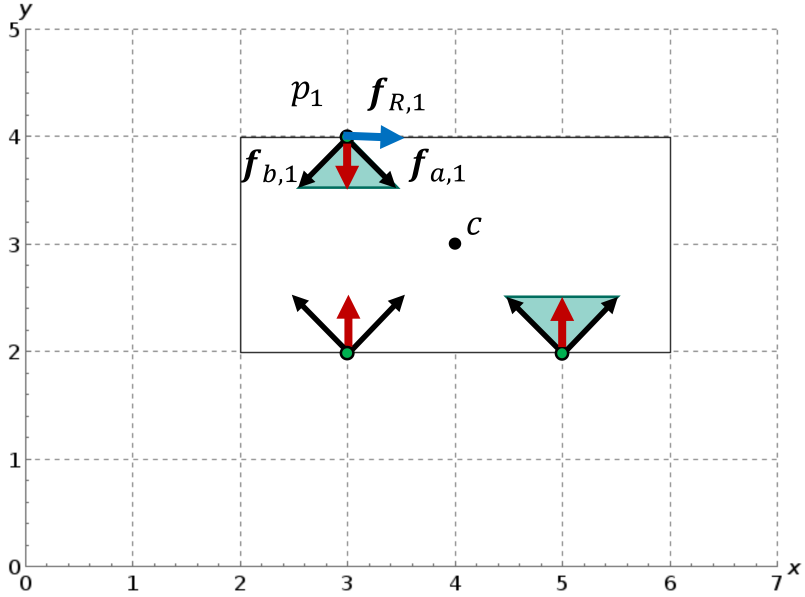


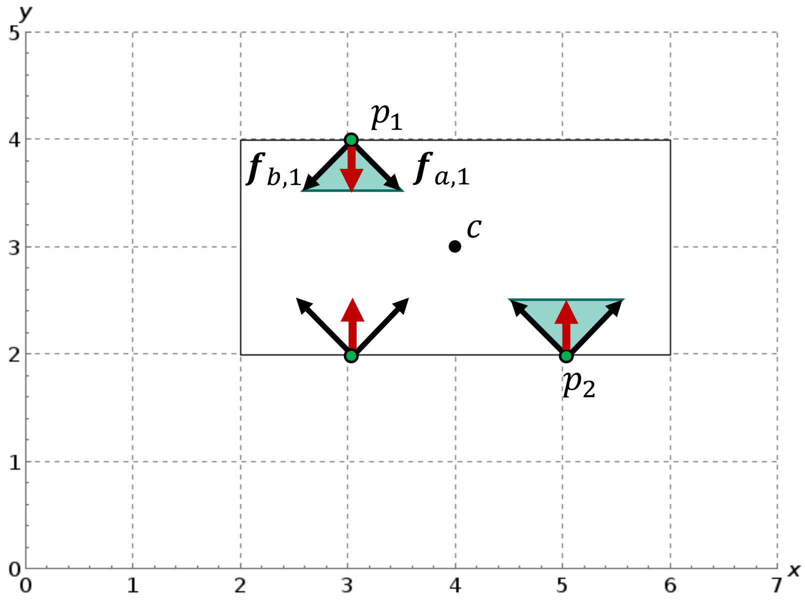
$$\boldsymbol{f}_{a,1} = \boldsymbol{f}_1 + \boldsymbol{f}_{R,1} = \begin{pmatrix} 0 \\ -0.5 \end{pmatrix} + \begin{pmatrix} 0.5 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5 \\ -0.5 \end{pmatrix}$$

$$\boldsymbol{f}_{b,1} = \boldsymbol{f}_1 - \boldsymbol{f}_{R,1} = \begin{pmatrix} 0 \\ -0.5 \end{pmatrix} - \begin{pmatrix} 0.5 \\ 0 \end{pmatrix} = \begin{pmatrix} -0.5 \\ -0.5 \end{pmatrix}$$

Exercise 1.3: Force Vectors at the Edges

$\boldsymbol{f}_R \perp \boldsymbol{f}, \|\boldsymbol{f}_R\| = \mu \cdot \|\boldsymbol{f}\|, \mu = 1$

$\boldsymbol{f}_2 = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$



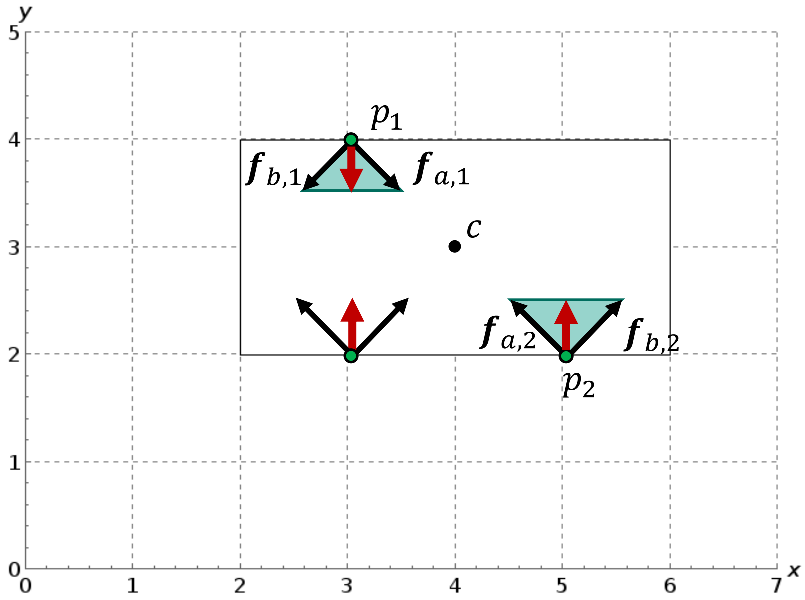
Exercise 1.3: Force Vectors at the Edges

$\boldsymbol{f}_R \perp \boldsymbol{f},\ \|\boldsymbol{f}_R\| = \mu \cdot \|\boldsymbol{f}\|,\ \mu = 1$

$\boldsymbol{f}_2 = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$

$\boldsymbol{f}_{\perp,2} = \begin{pmatrix} -0.5 \\ 0 \end{pmatrix}$

$\boldsymbol{f}_{R,2} = \mu \cdot \boldsymbol{f}_{\perp,2} = 1 \cdot \begin{pmatrix} -0.5 \\ 0 \end{pmatrix}$

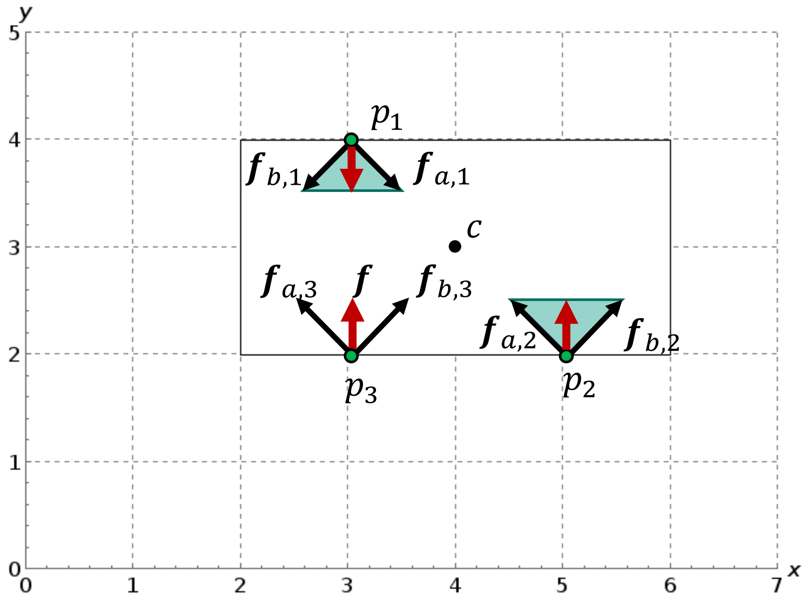


$$\boldsymbol{f}_{a,2} = \boldsymbol{f}_2 + \boldsymbol{f}_{R,2} = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix} + \begin{pmatrix} -0.5 \\ 0 \end{pmatrix} = \begin{pmatrix} -0.5 \\ 0.5 \end{pmatrix}$$
$$\boldsymbol{f}_{b,2} = \boldsymbol{f}_2 - \boldsymbol{f}_{R,2} = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix} - \begin{pmatrix} -0.5 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

Exercise 1.3: Force Vectors at the Edges

$\boldsymbol{f}_R \perp \boldsymbol{f},\ \|\boldsymbol{f}_R\| = \mu \cdot \|\boldsymbol{f}\|,\ \mu = 1$

$\boldsymbol{f}_3 = \boldsymbol{f}_2 = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$



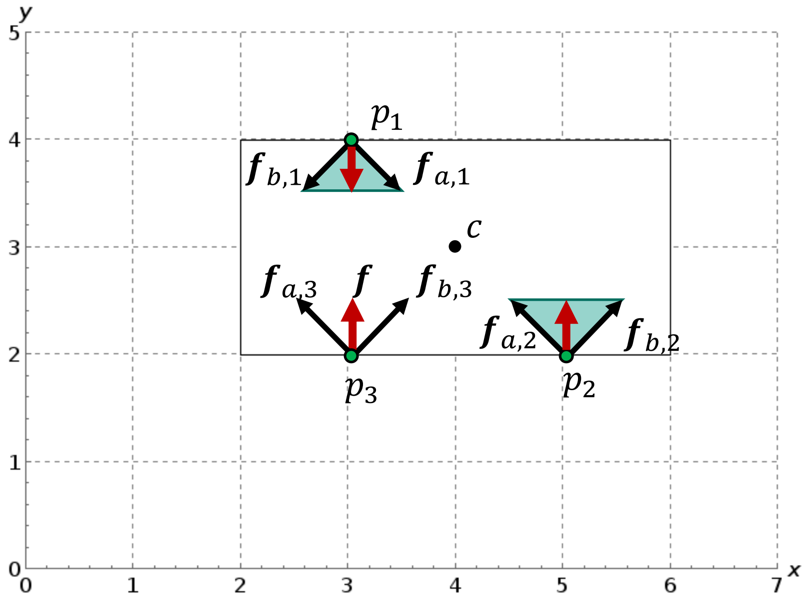
Exercise 1.3: Force Vectors at the Edges

$\boldsymbol{f}_R \perp \boldsymbol{f},\ \|\boldsymbol{f}_R\| = \mu \cdot \|\boldsymbol{f}\|,\ \mu = 1$

$\boldsymbol{f}_3 = \boldsymbol{f}_2 = \begin{pmatrix} 0 \\ 0.5 \end{pmatrix}$

$\boldsymbol{f}_{a,3} = \boldsymbol{f}_{a,2} = \begin{pmatrix} -0.5 \\ 0.5 \end{pmatrix}$

$\boldsymbol{f}_{b,3} = \boldsymbol{f}_{b,2} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$



Exercise 1.3: Bonus

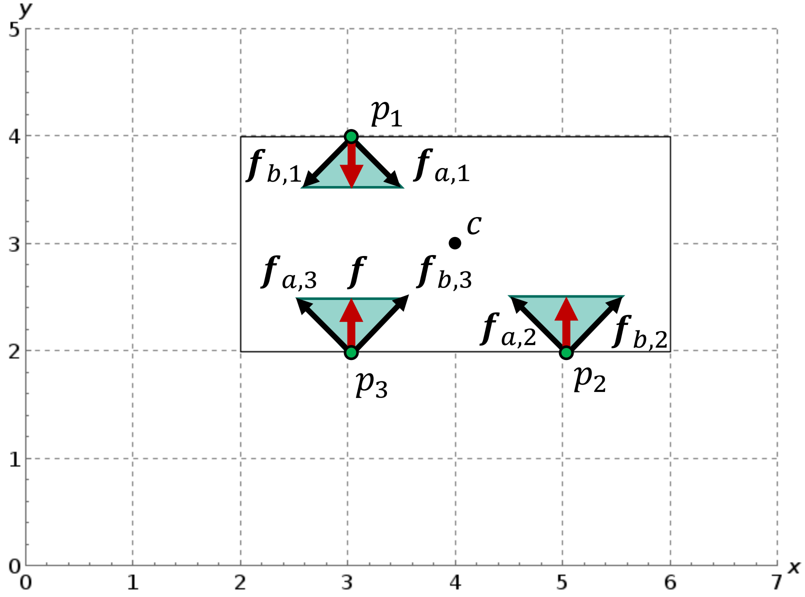
How does the friction triangle change when the friction coefficient decreases?

a) The width increases while the height remains the same.

b) The width decreases while the height remains the same.

c) Width and height decrease.

d) The height decreases, while the width remains the same.



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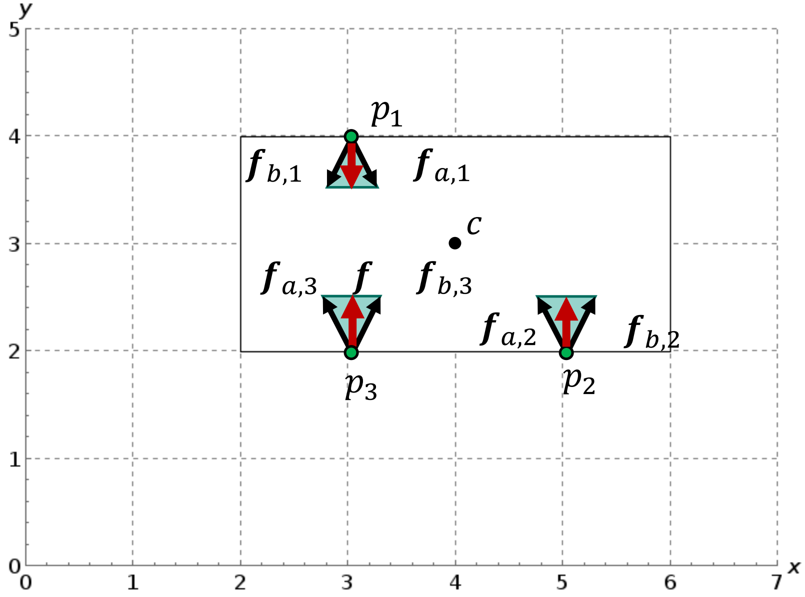
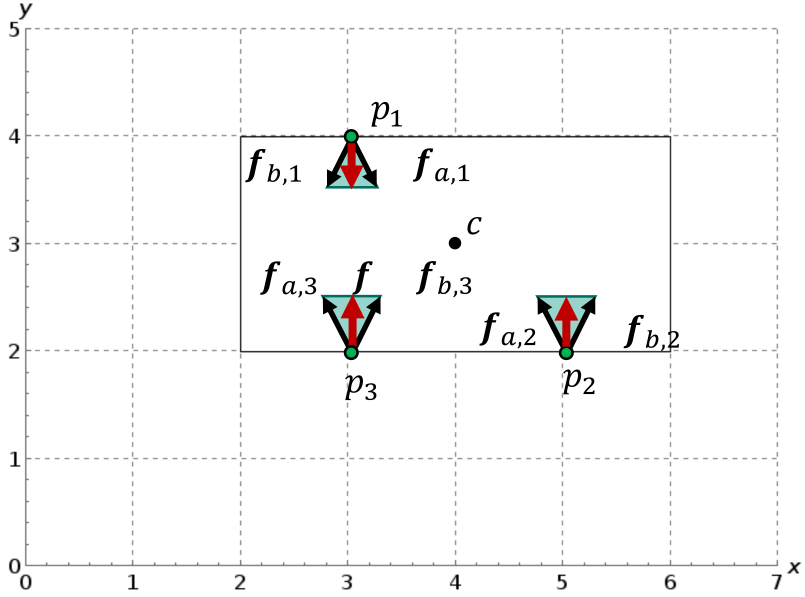


- The normal force $\boldsymbol{f}$ remains the same ➔ same heigth
- If μ decreases, $\|\boldsymbol{f}_R\| = \mu \cdot \|\boldsymbol{f}\|$ decreases as well ➔ width decreases

Exercise 1.3: Bonus

How can the original force of friction be restored despite a reduced friction coefficient?

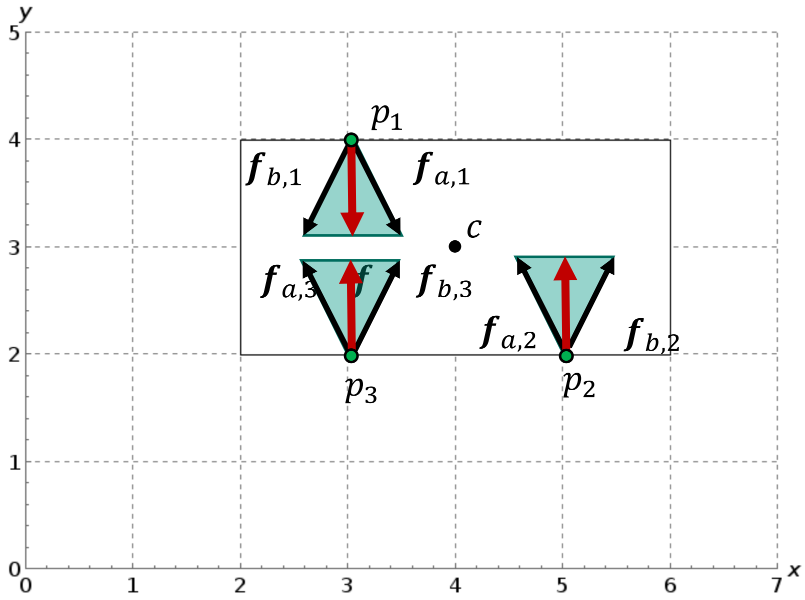


Exercise 1.3: Bonus

How can the original force of friction be restored despite a reduced friction coefficient?

→ **Increase the normal force *f***

Risk:
The object being grasped could be damaged



Exercise 1.3: Bonus

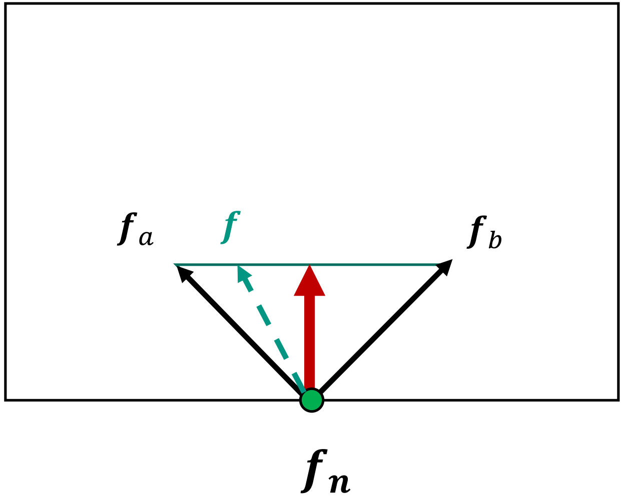
How can we describe the set of feasible force vectors using $\boldsymbol{f}_a$ and $\boldsymbol{f}_b$ assuming a fixed normal force $\boldsymbol{f}_{\boldsymbol{n}}$?

$\boldsymbol{f} = \boldsymbol{f}_a + \beta \cdot (\boldsymbol{f}_b - \boldsymbol{f}_a)$ with $\beta \in [0, 1]$

$\boldsymbol{f} = (1 - \beta) \cdot \boldsymbol{f}_a + \beta \cdot \boldsymbol{f}_b$

$\boldsymbol{f} = \mathrm{k}_1 \cdot \boldsymbol{f}_a + k_2 \cdot \boldsymbol{f}_b$

with $\mathrm{k}_1 + k_2 = 1$ and $\mathrm{k}_1 \geq 0;\ k_2 \geq 0$

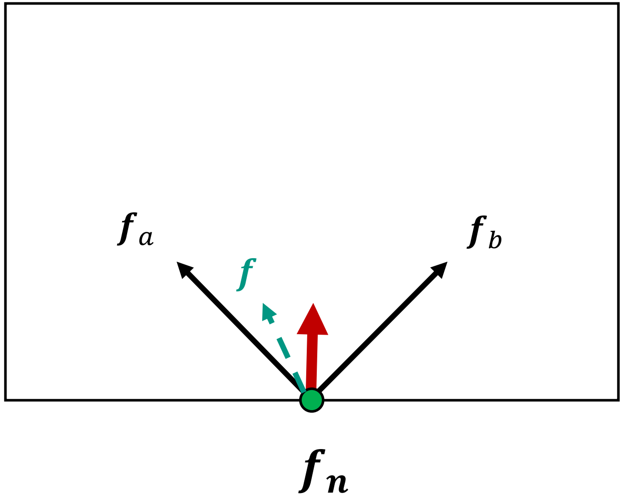


Exercise 1.3: Bonus

How can we describe the set of feasible force vectors using $\boldsymbol{f}_a$ and $\boldsymbol{f}_b$ assuming an arbitrary normal force $\boldsymbol{f}_{\boldsymbol{n}}$?

$\boldsymbol{f} = \mathrm{k}_1 \cdot \boldsymbol{f}_a + k_2 \cdot \boldsymbol{f}_b$

with $\mathrm{k}_1 \geq 0;\ k_2 \geq 0$

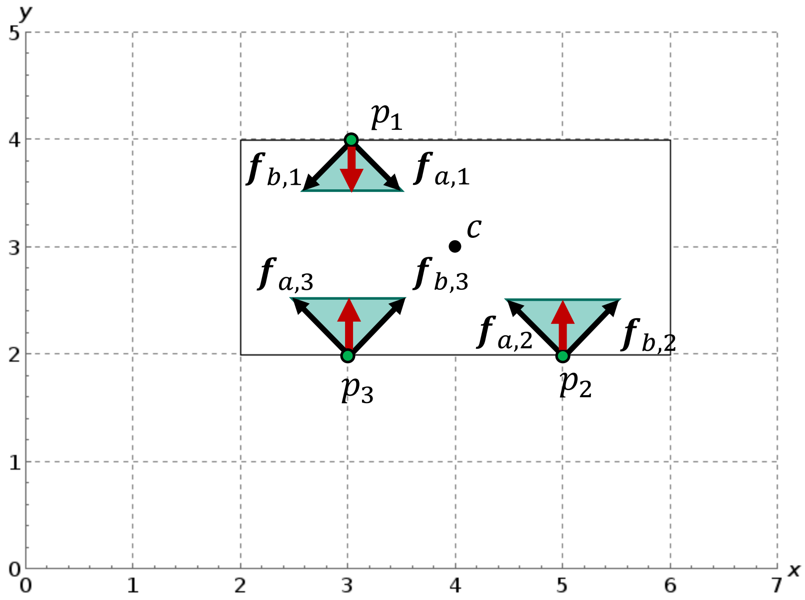


Exercise 2: Grasp Wrench Space

$$\boldsymbol{f}_{a,1} = \begin{pmatrix} 0.5 \\ -0.5 \end{pmatrix}, \ \boldsymbol{f}_{b,1} = \begin{pmatrix} -0.5 \\ -0.5 \end{pmatrix}$$

$$\boldsymbol{f}_{a,3} = \boldsymbol{f}_{a,2} = \begin{pmatrix} -0.5 \\ 0.5 \end{pmatrix}$$

$$\boldsymbol{f}_{b,3} = \boldsymbol{f}_{b,2} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$



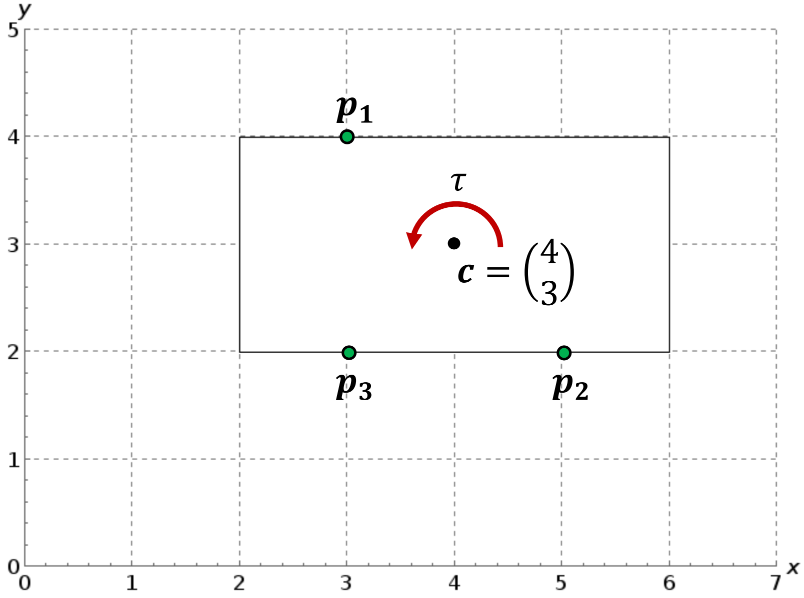
1. Wrenches at the contact points
2. Grasp Wrench Space for $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$
3. Grasp Wrench Space for $\boldsymbol{p}_1$, $\boldsymbol{p}_2$ and $\boldsymbol{p}_3$

Exercise 2.1: Compute Wrenches

- Wrenches in 2D: $w = \begin{pmatrix} f_x \\ f_y \\ \tau \end{pmatrix}$
- Torque in 2D:

$$\tau = \boldsymbol{d} \times \boldsymbol{f} = d_x \cdot f_y - d_y \cdot f_x$$

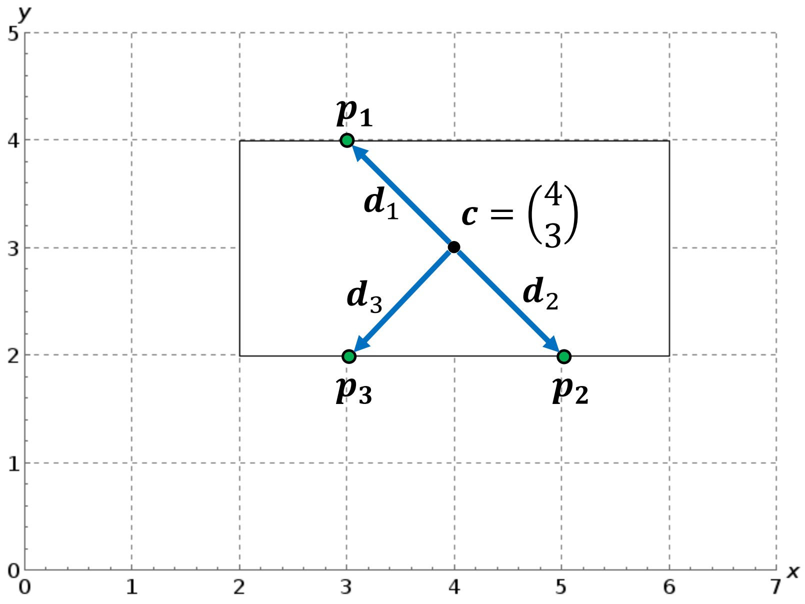
$$= \det \begin{pmatrix} d_x & f_x \\ d_y & f_y \end{pmatrix}$$



$$\boldsymbol{p_1} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \boldsymbol{p_2} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}, \boldsymbol{p_3} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

Exercise 2.1: Compute Wrenches

- Wrenches in 2D: $w = \begin{pmatrix} f_x \\ f_y \\ \tau \end{pmatrix}$
- Torque in 2D:
  $\tau = \boldsymbol{d} \times \boldsymbol{f} = d_x \cdot f_y - d_y \cdot f_x$



$\boldsymbol{p_1} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \boldsymbol{p_2} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}, \boldsymbol{p_3} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

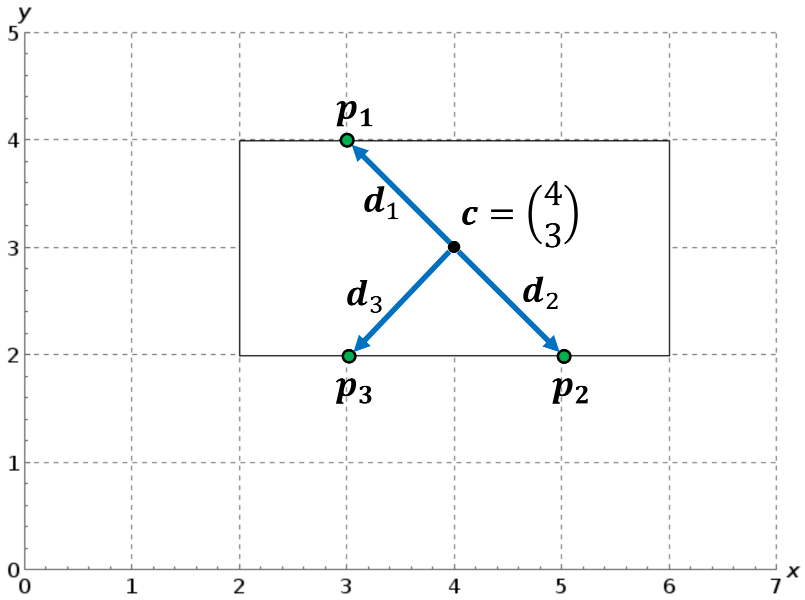
Exercise 2.1: Compute Wrenches

- Wrenches in 2D: $w = \begin{pmatrix} f_x \\ f_y \\ \tau \end{pmatrix}$
- Torque in 2D:
  $\tau = \boldsymbol{d} \times \boldsymbol{f} = d_x \cdot f_y - d_y \cdot f_x$

$$\boldsymbol{d}_1 = p_1 - c = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\boldsymbol{d}_2 = p_2 - c = \begin{pmatrix} 5 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\boldsymbol{d}_3 = p_3 - c = \begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

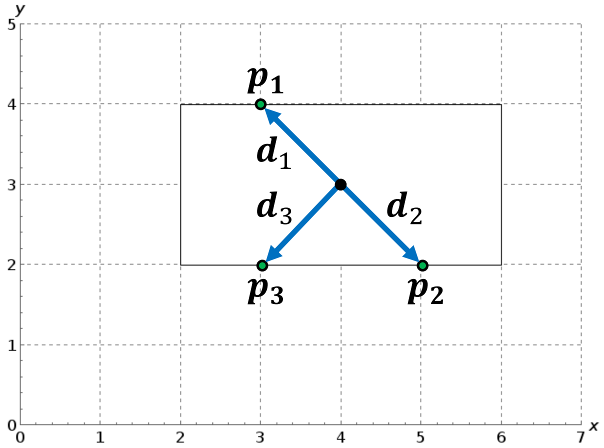


$\boldsymbol{p}_1 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, $\boldsymbol{p}_2 = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$, $\boldsymbol{p}_3 = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

Exercise 2.1: Compute Wrenches

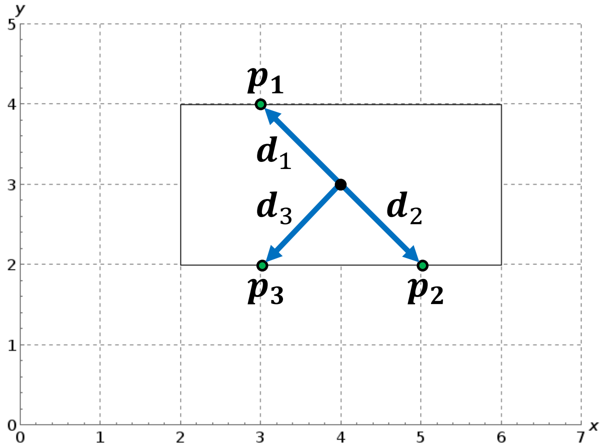
$\boldsymbol{d}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \boldsymbol{f}_{a,1} = \begin{pmatrix} 0.5 \\ -0.5 \end{pmatrix}, \boldsymbol{f}_{b,1} = \begin{pmatrix} -0.5 \\ -0.5 \end{pmatrix}$

$\tau_{a,1} =$



Exercise 2.1: Compute Wrenches

$$\boldsymbol{d}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \boldsymbol{f}_{a,1} = \begin{pmatrix} 0.5 \\ -0.5 \end{pmatrix}, \boldsymbol{f}_{b,1} = \begin{pmatrix} -0.5 \\ -0.5 \end{pmatrix}$$



$$\tau_{a,1} = \boldsymbol{d}_1 \times \boldsymbol{f}_{a,1}$$

$$= \begin{pmatrix} -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} 0.5 \\ -0.5 \end{pmatrix}$$

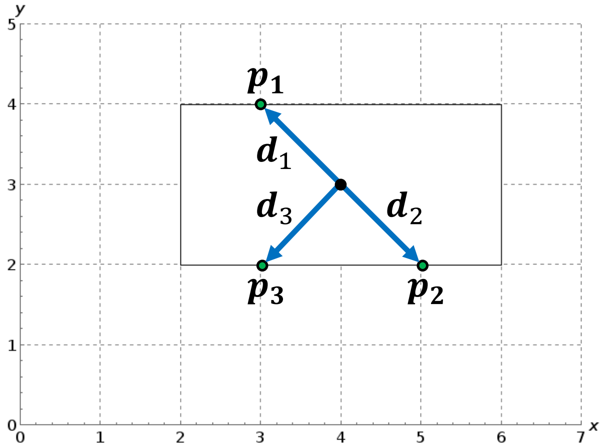
$$= (-1) \cdot (-0.5) - 1 \cdot 0.5$$

$$= 0.5 - 0.5 = 0$$

Exercise 2.1: Compute Wrenches

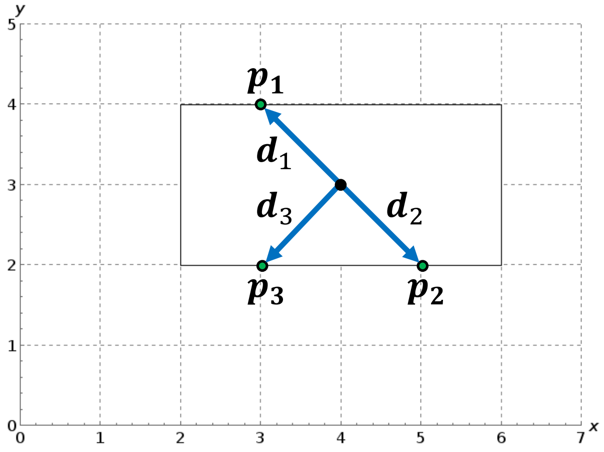
$\boldsymbol{d}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \boldsymbol{f}_{a,1} = \begin{pmatrix} 0.5 \\ -0.5 \end{pmatrix}, \boldsymbol{f}_{b,1} = \begin{pmatrix} -0.5 \\ -0.5 \end{pmatrix}$

$\tau_{b,1} =$



Exercise 2.1: Compute Wrenches

$$\boldsymbol{d}_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \boldsymbol{f}_{a,1} = \begin{pmatrix} 0.5 \\ -0.5 \end{pmatrix}, \boldsymbol{f}_{b,1} = \begin{pmatrix} -0.5 \\ -0.5 \end{pmatrix}$$



$$\tau_{b,1} = \boldsymbol{d}_1 \times \boldsymbol{f}_{b,1}$$

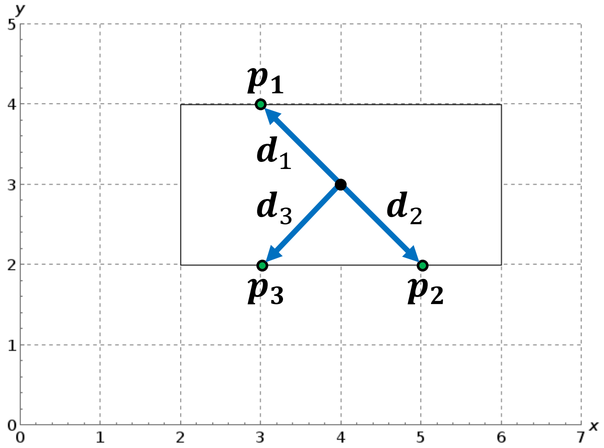
$$= \begin{pmatrix} -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} -0.5 \\ -0.5 \end{pmatrix}$$

$$= (-1) \cdot (-0.5) - 1 \cdot (-0.5)$$

$$= 0.5 + 0.5 = 1$$

Exercise 2.1: Compute Wrenches

$$\boldsymbol{d}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \boldsymbol{f}_{a,2} = \begin{pmatrix} -0.5 \\ 0.5 \end{pmatrix}, \boldsymbol{f}_{b,2} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$



$$\tau_{a,2} =$$

$$\tau_{b,2} =$$

Exercise 2.1: Compute Wrenches

$$\boldsymbol{d}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \boldsymbol{f}_{a,2} = \begin{pmatrix} -0.5 \\ 0.5 \end{pmatrix}, \boldsymbol{f}_{b,2} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

$$\tau_{a,2} = \boldsymbol{d}_2 \times \boldsymbol{f}_{a,2} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \times \begin{pmatrix} -0.5 \\ 0.5 \end{pmatrix}$$

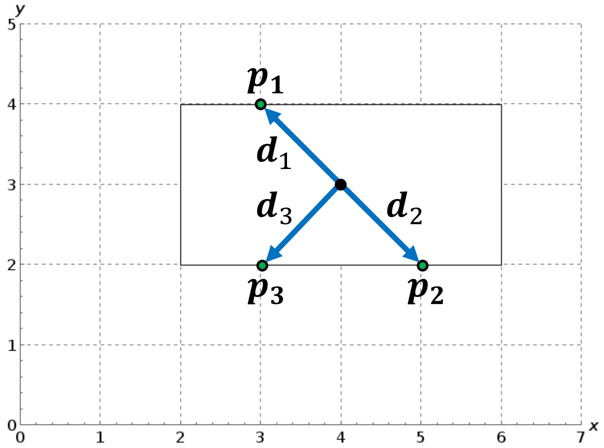
$$= 1 \cdot 0.5 - (-1) \cdot (-0.5)$$

$$= 0.5 - 0.5 = 0$$

$$\tau_{b,2} = \boldsymbol{d}_2 \times \boldsymbol{f}_{b,2} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \times \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

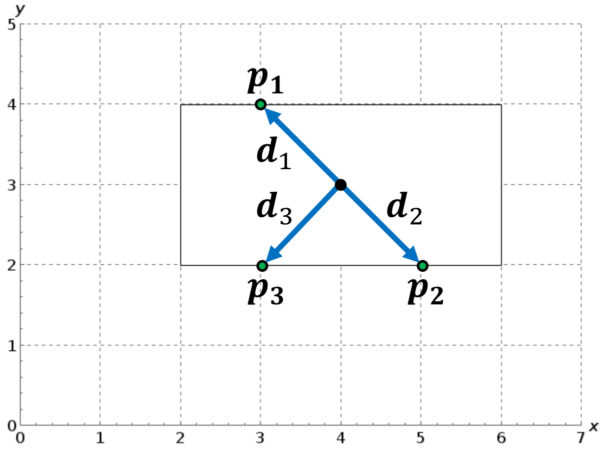
$$= 1 \cdot 0.5 - (-1) \cdot 0.5$$

$$= 0.5 + 0.5 = 1$$



Exercise 2.1: Compute Wrenches

$$\boldsymbol{d}_3 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \boldsymbol{f}_{a,3} = \begin{pmatrix} -0.5 \\ 0.5 \end{pmatrix}, \boldsymbol{f}_{b,3} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$



$$\tau_{a,3} =$$

$$\tau_{b,3} =$$

Exercise 2.1: Compute Wrenches

$$\boldsymbol{d}_3 = \begin{pmatrix}-1\\-1\end{pmatrix}, \boldsymbol{f}_{a,3} = \begin{pmatrix}-0.5\\0.5\end{pmatrix}, \boldsymbol{f}_{b,3} = \begin{pmatrix}0.5\\0.5\end{pmatrix}$$

$$\tau_{a,3} = \boldsymbol{d}_3 \times \boldsymbol{f}_{a,3} = \begin{pmatrix}-1\\-1\end{pmatrix} \times \begin{pmatrix}-0.5\\0.5\end{pmatrix}$$

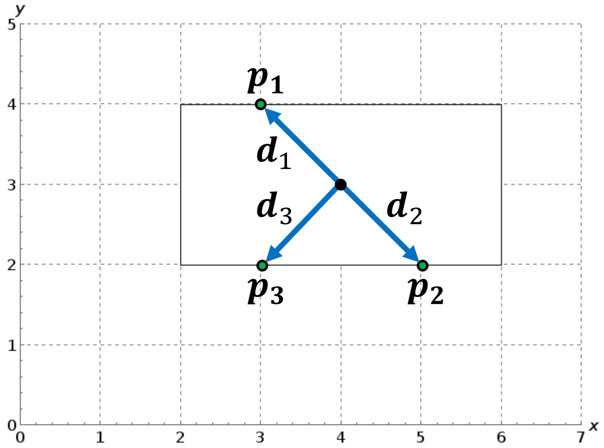
$$= (-1) \cdot 0.5 - (-1) \cdot (-0.5)$$

$$= -0.5 - 0.5 = -1$$

$$\tau_{b,3} = \boldsymbol{d}_3 \times \boldsymbol{f}_{b,3} = \begin{pmatrix}-1\\-1\end{pmatrix} \times \begin{pmatrix}0.5\\0.5\end{pmatrix}$$

$$= (-1) \cdot 0.5 - (-1) \cdot 0.5$$

$$= -0.5 + 0.5 = 0$$



Exercise 2.1: Compute Wrenches

$$\boldsymbol{f}_{a,1} = \begin{pmatrix} 0.5 \\ -0.5 \end{pmatrix}, \ \boldsymbol{f}_{b,1} = \begin{pmatrix} -0.5 \\ -0.5 \end{pmatrix}$$

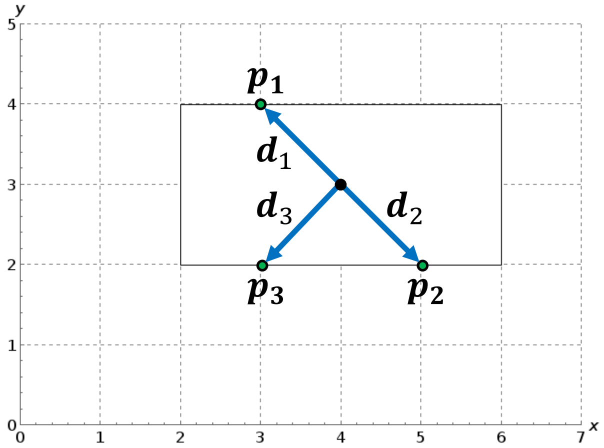
$$\boldsymbol{f}_{a,3} = \boldsymbol{f}_{a,2} = \begin{pmatrix} -0.5 \\ 0.5 \end{pmatrix}$$

$$\boldsymbol{f}_{b,3} = \boldsymbol{f}_{b,2} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

$$\tau_{a,1} = 0, \qquad \tau_{b,1} = 1$$

$$\tau_{a,2} = 0, \qquad \tau_{b,2} = 1$$

$$\tau_{a,3} = -1, \qquad \tau_{b,3} = 0$$



Exercise 2.1: Compute Wrenches

$$\boldsymbol{f}_{a,1} = \begin{pmatrix} 0.5 \\ -0.5 \end{pmatrix},\ \boldsymbol{f}_{b,1} = \begin{pmatrix} -0.5 \\ -0.5 \end{pmatrix}$$

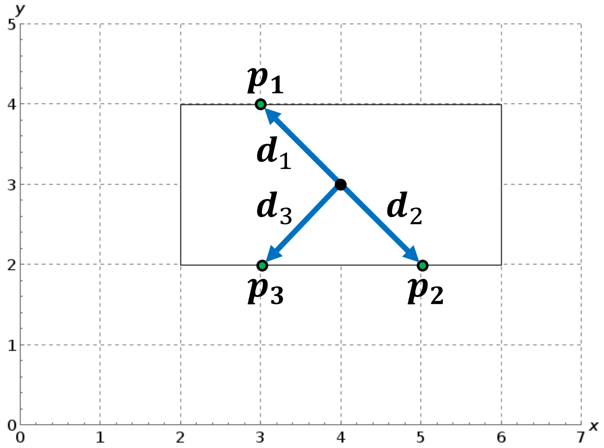
$$\boldsymbol{f}_{a,3} = \boldsymbol{f}_{a,2} = \begin{pmatrix} -0.5 \\ 0.5 \end{pmatrix}$$

$$\boldsymbol{f}_{b,3} = \boldsymbol{f}_{b,2} = \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$$

$$\tau_{a,1} = 0, \qquad \tau_{b,1} = 1$$

$$\tau_{a,2} = 0, \qquad \tau_{b,2} = 1$$

$$\tau_{a,3} = -1, \qquad \tau_{b,3} = 0$$



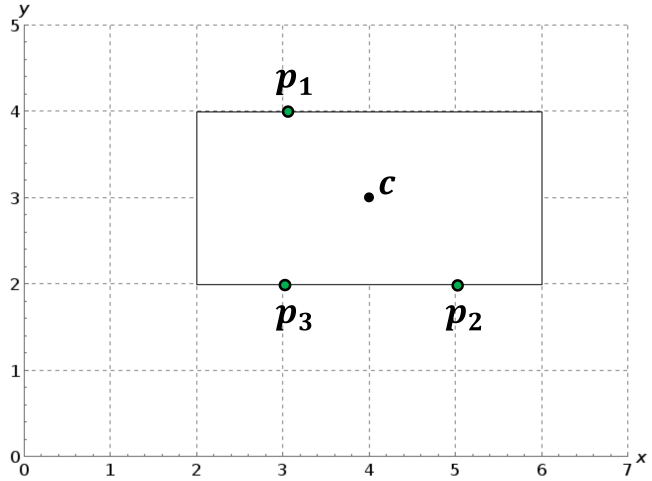
$$w_{a,1} = (\boldsymbol{f}_{a,1}, \tau_{a,1}) = (0.5, -0.5, 0) \qquad w_{b,1} = (\boldsymbol{f}_{b,1}, \tau_{b,1}) = (-0.5, -0.5, 1)$$

$$w_{a,2} = (\boldsymbol{f}_{a,2}, \tau_{a,2}) = (-0.5, 0.5, 0) \qquad w_{b,2} = (\boldsymbol{f}_{b,2}, \tau_{b,2}) = (0.5, 0.5, 1)$$

$$w_{a,3} = (\boldsymbol{f}_{a,3}, \tau_{a,3}) = (-0.5, 0.5, -1) \qquad w_{b,2} = (\boldsymbol{f}_{b,3}, \tau_{b,3}) = (0.5, 0.5, 0)$$

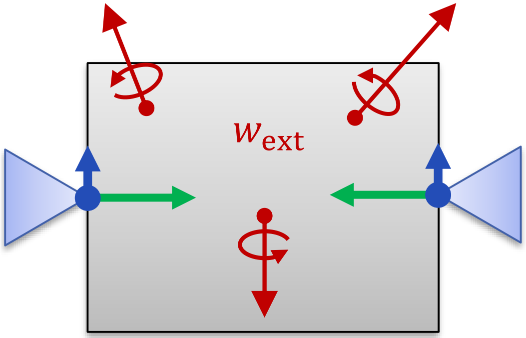
Exercise 2.2: Grasp Wrench Space for 2 Contact Points

- Draw the projection of the Grasp Wrench Space onto the (f_y, τ) plane for the contact points $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$



Force-closed Grasps

- **Question:** Can a grasp counteract **any external wrenches**?



- **Assumption:** Contacts can exert arbitrarily large forces.
  → We can multiply each wrench $\boldsymbol{w}_{\mathbf{ij}}$ with an arbitrary factor $\boldsymbol{k}_{\mathbf{ij}} > 0$.
- Resulting question: Can the grasp generate **arbitrary wrenches**?
  → If so, the grasp can generate $-w_{\text{ext}}$ and the grasp is force-closed.

Force-closed Grasps

- Grasp matrix (2D)

$$G = \left[\boldsymbol{w}_{a,1}, \boldsymbol{w}_{b,1}, \boldsymbol{w}_{a,2}, \boldsymbol{w}_{b,2}, \dots, \boldsymbol{w}_{a,m}, \boldsymbol{w}_{b,m}\right] \in \mathbb{R}^{3\times 2m}$$

- A grasp is force-closed, if it can counteract **any external wrench** w_{ext}:

$$\forall\, \boldsymbol{w}_{\text{ext}} = \left(f_x, f_y, \tau\right) \in \mathbb{R}^3:$$

$$\exists\, \boldsymbol{k} \in R^{2m},\ \boldsymbol{k} \geq \boldsymbol{0}:$$

$$G \cdot \boldsymbol{k} + \boldsymbol{w}_{\text{ext}} = 0$$

Force-closed Grasps

Karlsruher Institut für Technologie

- Grasp matrix (2D)

$$G = \left[\boldsymbol{w}_{a,1}, \boldsymbol{w}_{b,1}, \boldsymbol{w}_{a,2}, \boldsymbol{w}_{b,2}, \dots, \boldsymbol{w}_{a,m}, \boldsymbol{w}_{b,m}\right] \in \mathbb{R}^{3\times 2m}$$

- A grasp is force-closed, if it can counteract **any external wrench** w_{ext}:

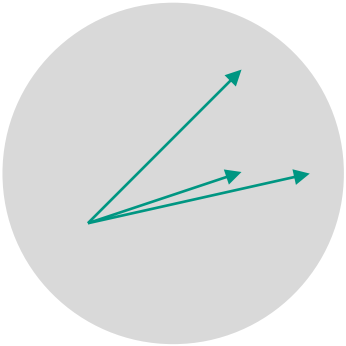
$$\forall\, \boldsymbol{w}_{\text{ext}} = \left(f_x, f_y, \tau\right) \in \mathbb{R}^3:$$

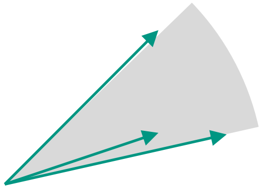
$$\exists\, \boldsymbol{k} \in R^{2m},\ \boldsymbol{k} \geq \boldsymbol{0}:$$

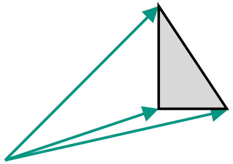
$$G \cdot \boldsymbol{k} + \boldsymbol{w}_{\text{ext}} = 0$$

$$\text{pos}(G) = \mathbb{R}^3$$

Linear Hull and Convex Hull







Linear Hull

$$\mathrm{span}(A) = \left\{ \sum_{i=1}^{j} k_i \cdot a_i \;\middle|\; k_i \in \mathbb{R} \right\}$$

Positive linear Hull

$$\mathrm{pos}(A) = \left\{ \sum_{i=1}^{j} k \cdot a_i \;\middle|\; k_i \geq 0 \right\}$$

Convex Hull

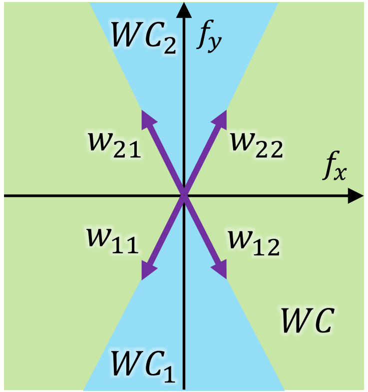
$$\mathrm{conv}(A) = \left\{ \sum_{i=1}^{j} k_i \cdot a_i \;\middle|\; k_i \geq 0 \text{ and } \sum_i k_i = 1 \right\}$$

$$\mathrm{conv}(A) \subseteq \mathrm{pos}(A) \subseteq \mathrm{span}(A)$$

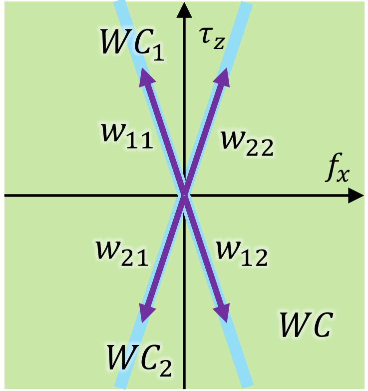
Which Wrenches can be generated: Example in 2D

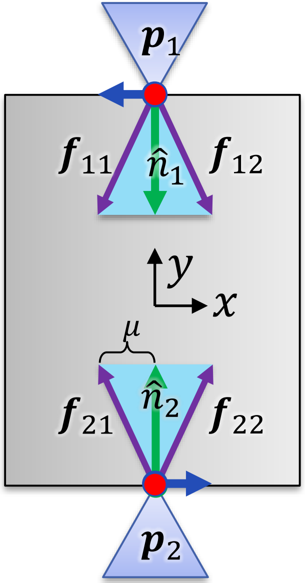
- Wrenches (edges of the friction cones):
  - $\boldsymbol{w}_{11} = (-\mu \quad -1 \quad 3\mu)^T, \qquad \boldsymbol{w}_{12} = (\mu \quad -1 \quad -3\mu)^T$
  - $\boldsymbol{w}_{21} = (-\mu \quad 1 \quad -3\mu)^T, \qquad \boldsymbol{w}_{22} = (\mu \quad 1 \quad 3\mu)^T$
- Projections of the 3D Wrench space onto subspaces:

f_x, f_y-Plane:



f_x, τ_z-Plane:



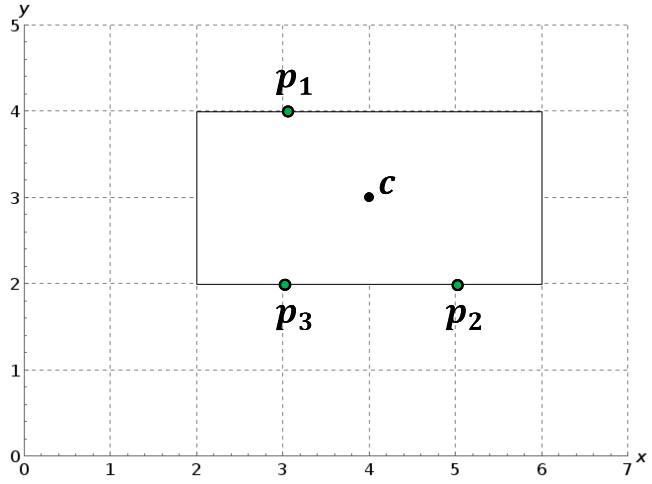


- If the total set of wrenches spans $\mathbb{R}^3 \Rightarrow$ **Wrenches can be created in all directions**.
- $\Rightarrow$ **Grasp is force-closed.**

Exercise 2.2: Grasp Wrench Space for 2 Contact Points

- Draw the projection of the **Grasp Wrench Space** onto the $(\boldsymbol{f}_y, \boldsymbol{\tau})$ plane for the contact points $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$

- What is the **Grasp Wrench Space?**

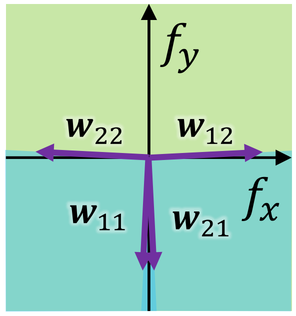


Grasp-Wrench-Space (2D)

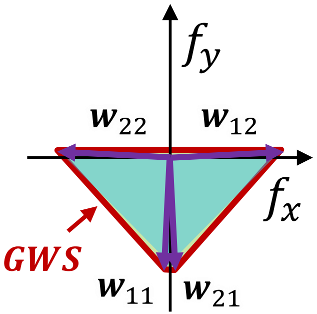
Let $\boldsymbol{w}_1, \dots, \boldsymbol{w}_m \in \mathbb{R}^3$ be the wrenches of the friction triangles of all contacts.

The **Grasp-Wrench-Space** GWS is the convex hull of the $\boldsymbol{w}_i$

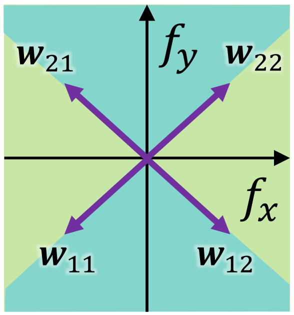
$$GWS = \text{conv}(\{\boldsymbol{w}_i\}) = \left\{ \sum_{i=1}^{m} k_i \boldsymbol{w}_i \mid k_i \geq 0 \text{ and } \sum_{i=1}^{m} k_i = 1 \right\}$$



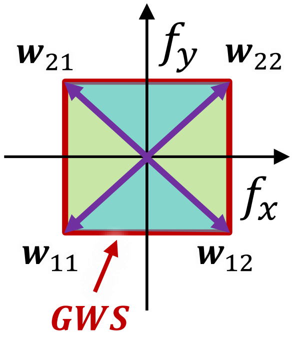
linear hull



convex hull



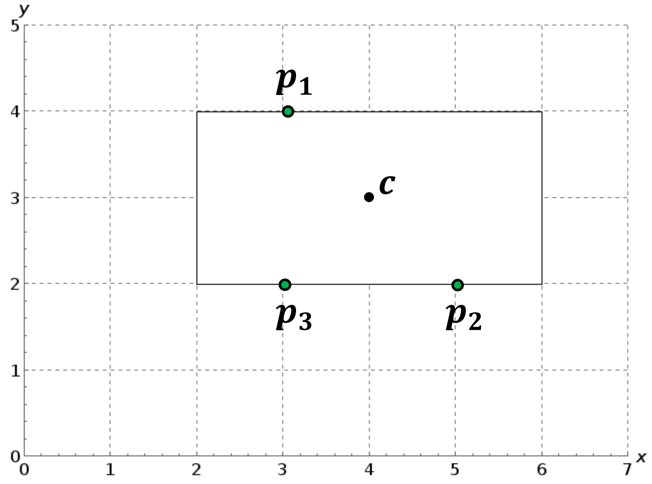
linear hull



convex hull

Exercise 2.2: Grasp Wrench Space for 2 Contact Points

- Draw the projection of the **Grasp Wrench Space** onto the (f_y, τ) plane for the contact points $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$



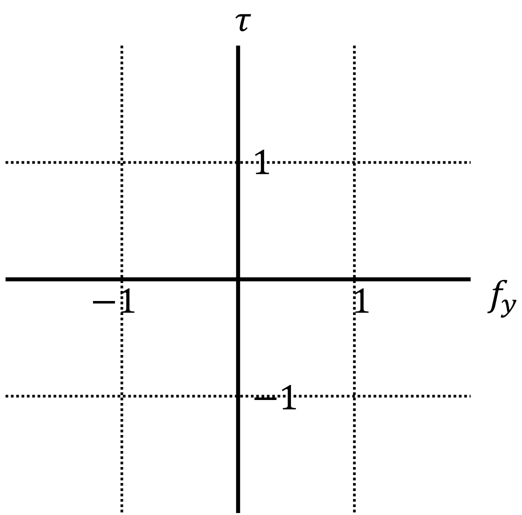
$$w_{a,1} = (0.5,\ \ -0.5,\ \ 0) \qquad w_{b,1} = (-0.5, -0.5,\ \ 1)$$
$$w_{a,2} = (-0.5,\ \ 0.5,\ \ 0) \qquad w_{b,2} = (0.5,\ \ \ 0.5,\ \ 1)$$

Exercise 2.2: Grasp Wrench Space for 2 Contact Points

$w_{a,1} = (0.5, \ -0.5, \ 0)$ $\quad$ $w_{b,1} = (-0.5, -0.5, \ 1)$

$w_{a,2} = (-0.5, \ 0.5, \ 0)$ $\quad$ $w_{b,2} = (0.5, \ 0.5, \ 1)$

- Projection of the Grasp Wrench Space onto (f_y, τ) for $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$

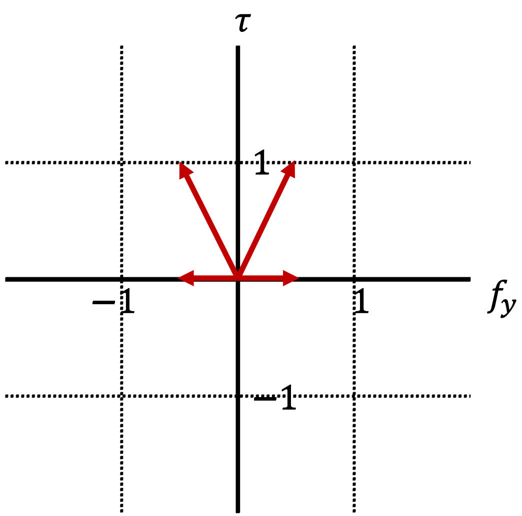


Exercise 2.2: Grasp Wrench Space for 2 Contact Points

$w_{a,1} = (0.5, \ -0.5, \ 0)$ $\qquad$ $w_{b,1} = (-0.5, -0.5, \ 1)$

$w_{a,2} = (-0.5, \ 0.5, \ 0)$ $\qquad$ $w_{b,2} = (0.5, \ 0.5, \ 1)$

- Projection of the Grasp Wrench Space onto (f_y, τ) for $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$

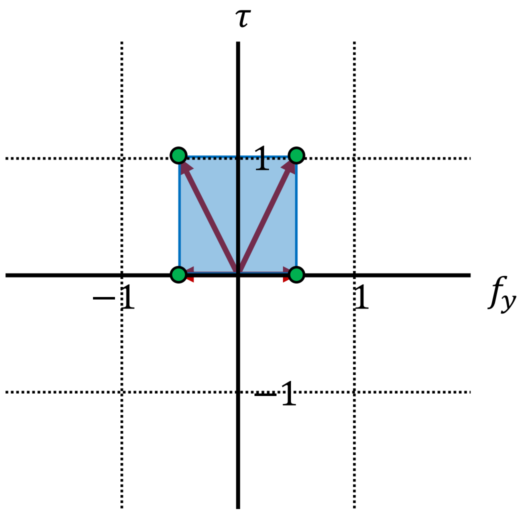


Exercise 2.2: Grasp Wrench Space for 2 Contact Points

$w_{a,1} = (0.5, \quad -0.5, \quad 0)$ $\qquad$ $w_{b,1} = (-0.5, -0.5, \quad 1)$

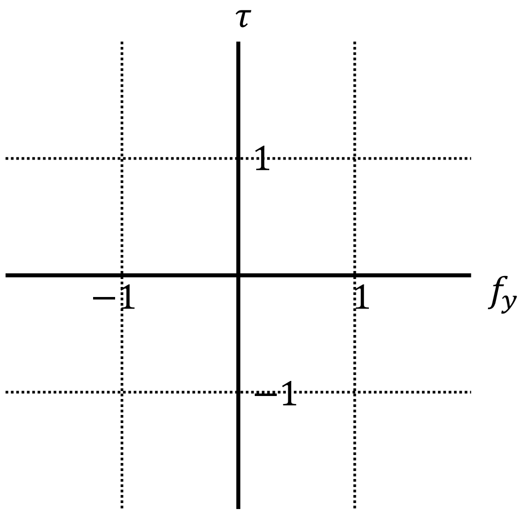
$w_{a,2} = (-0.5, \quad 0.5, \quad 0)$ $\qquad$ $w_{b,2} = (0.5, \quad 0.5, \quad 1)$

- Projection of the Grasp Wrench Space onto (f_y, τ) for $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$



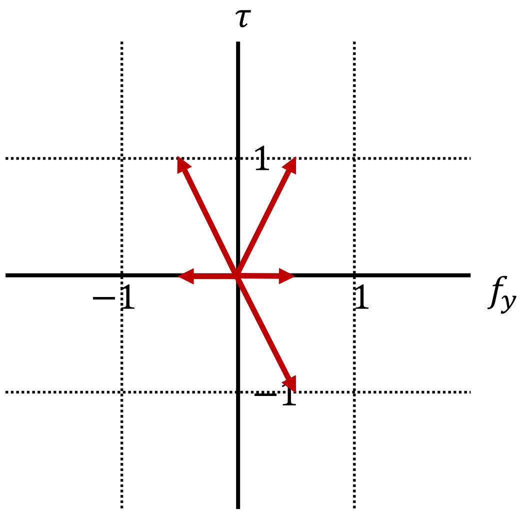
Exercise 2.3: Grasp Wrench Space for 3 Contact Points

$$w_{a,1} = (0.5, \quad -0.5, \quad 0) \qquad w_{b,1} = (-0.5, -0.5, \quad 1)$$
$$w_{a,2} = (-0.5, \quad 0.5, \quad 0) \qquad w_{b,2} = (0.5, \quad 0.5, \quad 1)$$
$$w_{a,3} = (-0.5, \quad 0.5, -1) \qquad w_{b,3} = (0.5, \quad 0.5, \quad 0)$$



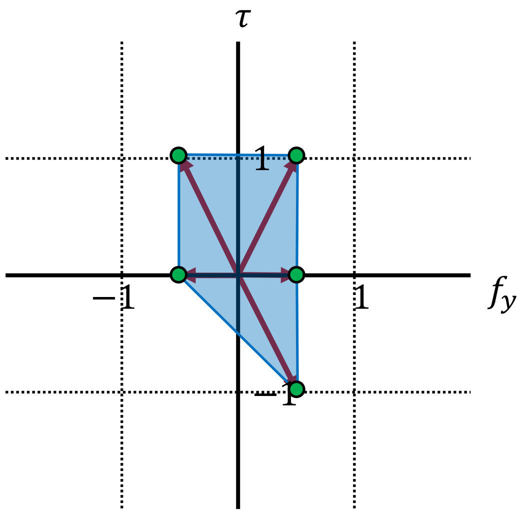
Exercise 2.3: Grasp Wrench Space for 3 Contact Points

$$w_{a,1} = (0.5, \; -0.5, \; 0) \qquad w_{b,1} = (-0.5, -0.5, \; 1)$$
$$w_{a,2} = (-0.5, \; 0.5, \; 0) \qquad w_{b,2} = (0.5, \; 0.5, \; 1)$$
$$w_{a,3} = (-0.5, \; 0.5, -1) \qquad w_{b,3} = (0.5, \; 0.5, \; 0)$$



Exercise 2.3: Grasp Wrench Space for 3 Contact Points

$$w_{a,1} = (0.5, \ -0.5, \ 0) \qquad w_{b,1} = (-0.5, -0.5, \ 1)$$
$$w_{a,2} = (-0.5, \ 0.5, \ 0) \qquad w_{b,2} = (0.5, \ 0.5, \ 1)$$
$$w_{a,3} = (-0.5, \ 0.5, -1) \qquad w_{b,3} = (0.5, \ 0.5, \ 0)$$

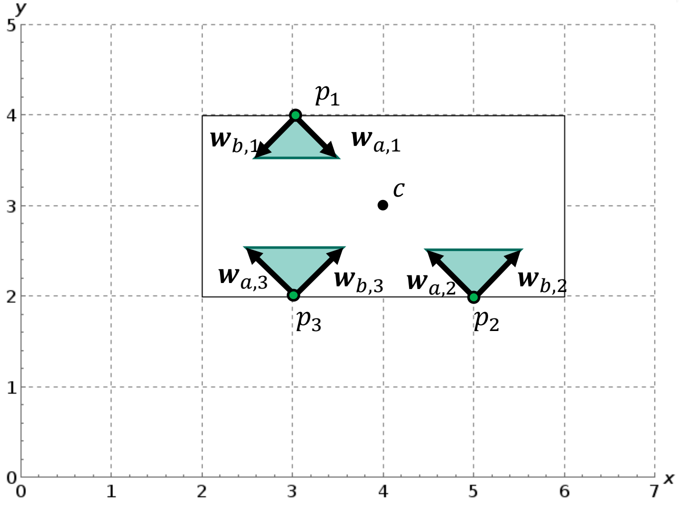


Exercise 3: Force Closure

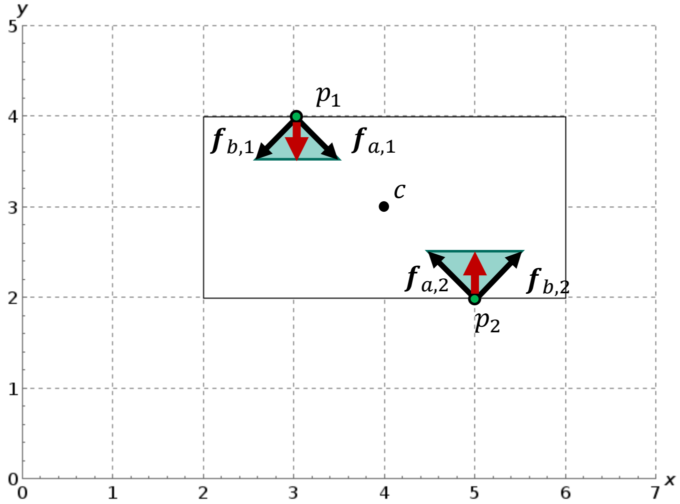
- Are the following grasps force-closed?

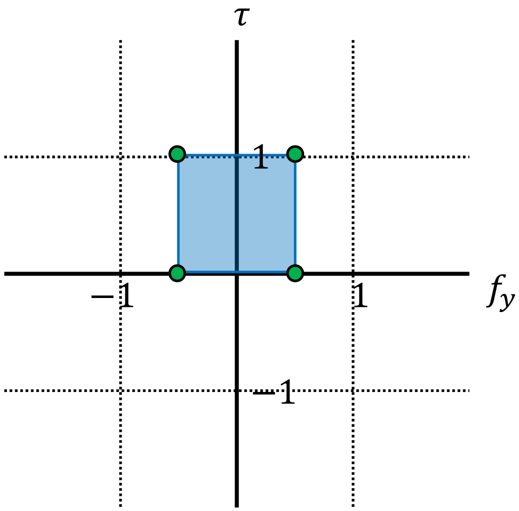
  1. Two-finger grasp: p_1, p_2
  2. Three-finger grasp: p_1, p_2 and p_3

- How would you calculate the ε-metric for the two grasps?



Exercise 3: Two-finger grasp



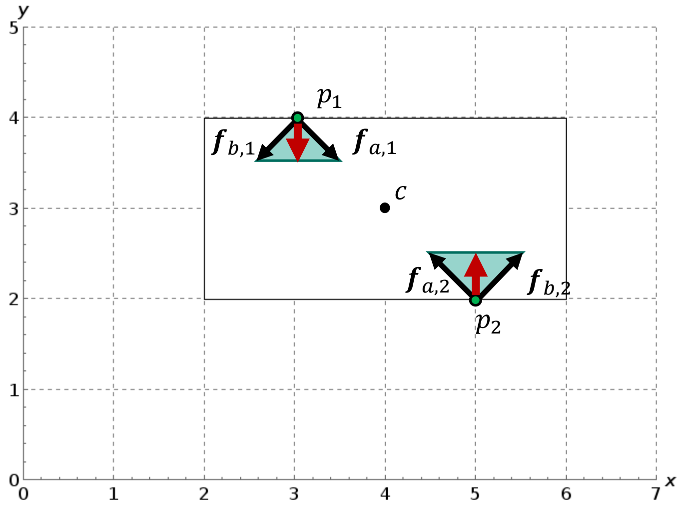


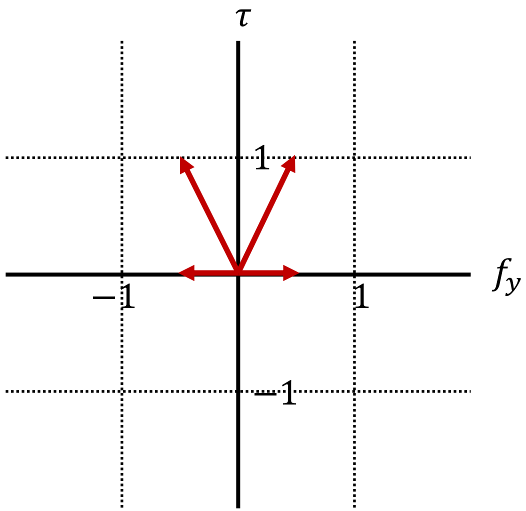
Is the two-finger grasp **force-closed**?

$\exists\, \boldsymbol{k} \in \mathbb{R}^m, \boldsymbol{k} \geq \boldsymbol{0} \quad : \quad \boldsymbol{G} \cdot \boldsymbol{k} + \boldsymbol{w}_{\text{ext}} = \boldsymbol{0}$

Exercise 3: Two-finger grasp

- What is $\text{pos}(G)$?

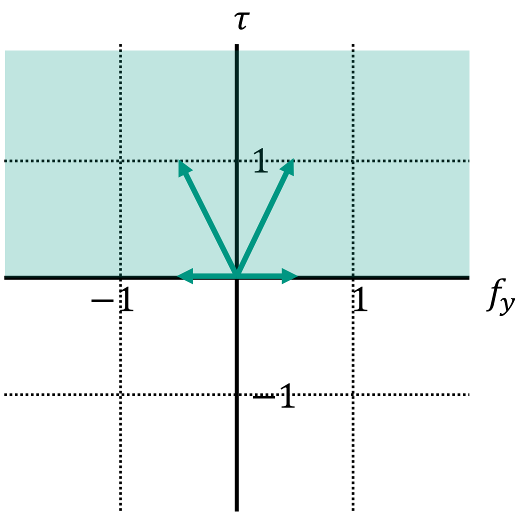




Exercise 3: Two-finger grasp

- What is $\text{pos}(G)$?

$$\text{pos}(G') = \left\{ \begin{pmatrix} f_x \\ f_y \\ \tau \end{pmatrix} \in \mathbb{R}^3 \;\middle|\; \tau \geq 0 \right\} \neq \mathbb{R}^3$$

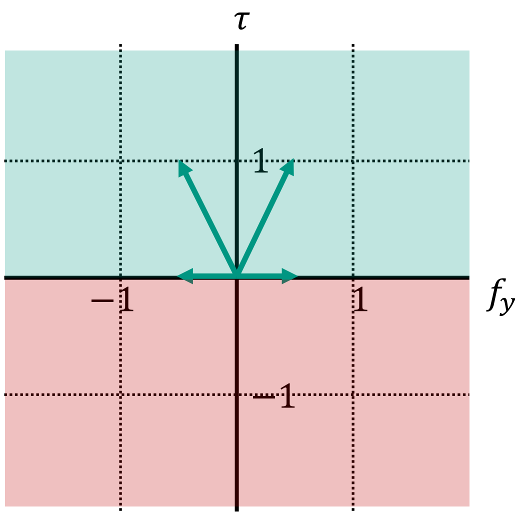


Exercise 3: Two-finger grasp

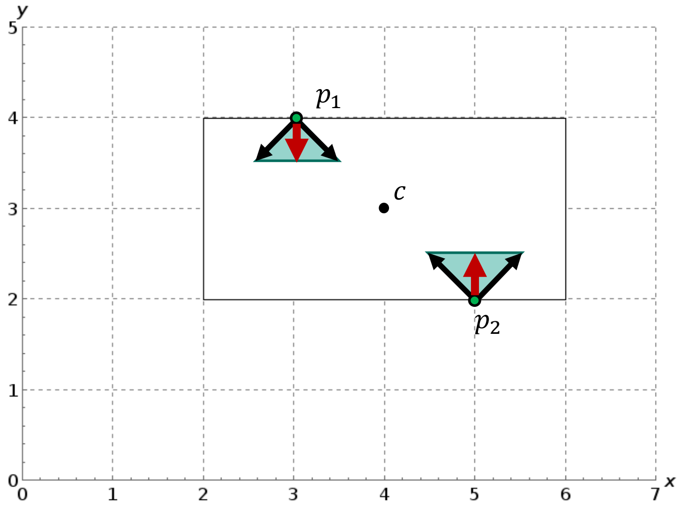
- What is $\text{pos}(G)$?

$$\text{pos}(G) = \left\{ \begin{pmatrix} f_x \\ f_y \\ \tau \end{pmatrix} \in \mathbb{R}^3 \;\middle|\; \tau \geq 0 \right\} \neq \mathbb{R}^3$$

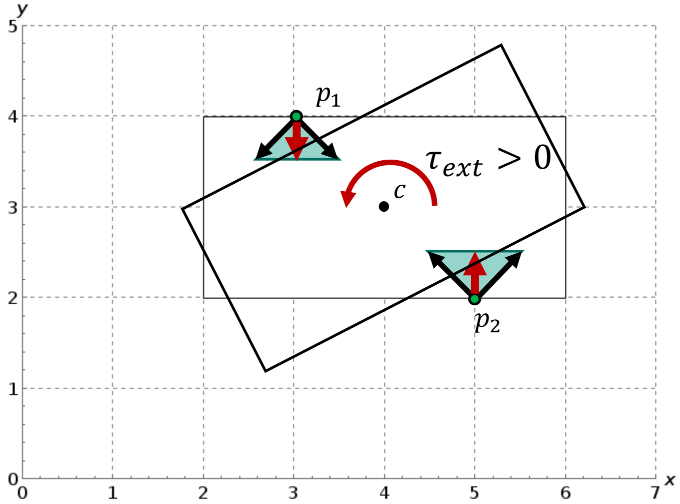
- The grasp cannot generate torques $\tau < 0$



Exercise 3: Two-finger grasp

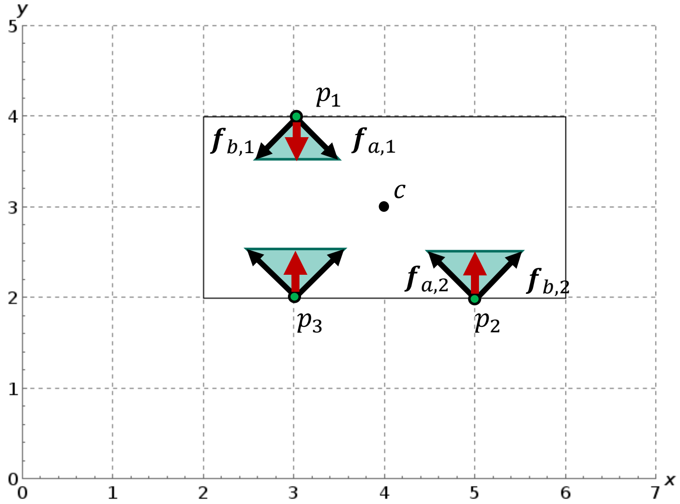


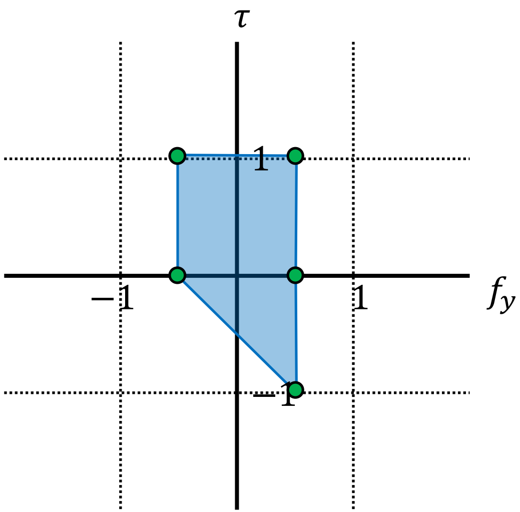
Exercise 3: Two-finger grasp



- The grasp cannot generate torques $\tau < 0$
- The grasp cannot counteract external torques $\tau_{ext} > 0$

Exercise 3: Three-finger grasp



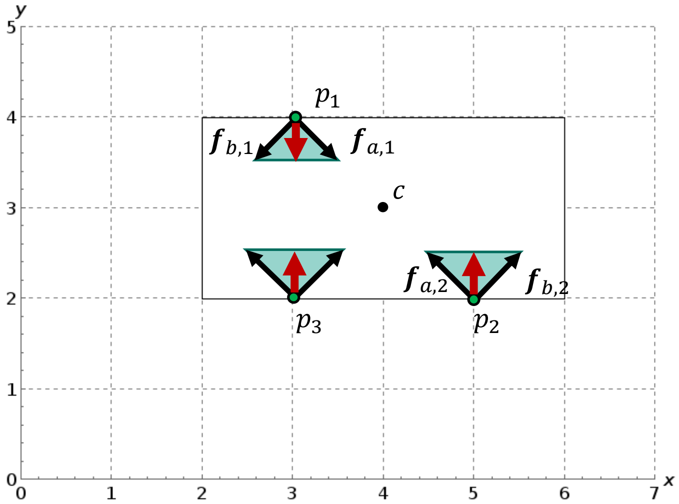


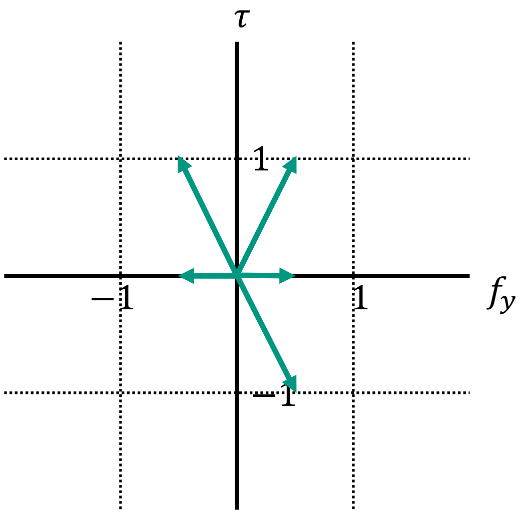
Is the grasp **force-closed**?

$$\exists\, \boldsymbol{k} \in \mathbb{R}^m, \boldsymbol{k} \geq \boldsymbol{0} \quad : \quad \boldsymbol{G} \cdot \boldsymbol{k} + \boldsymbol{w}_{\text{ext}} = \boldsymbol{0}$$

Exercise 3: Three-finger grasp

- What is $\text{pos}(G)$?

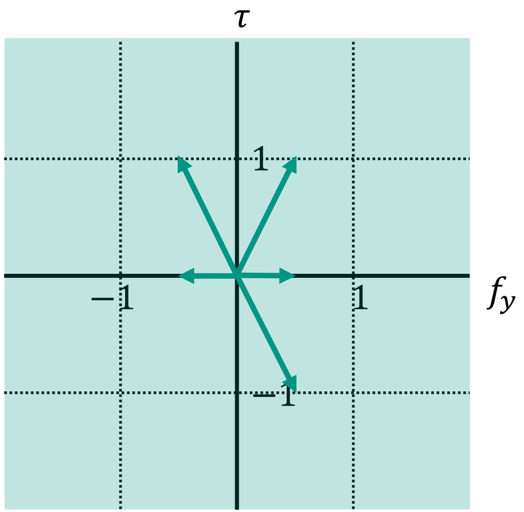




Exercise 3: Three-finger grasp

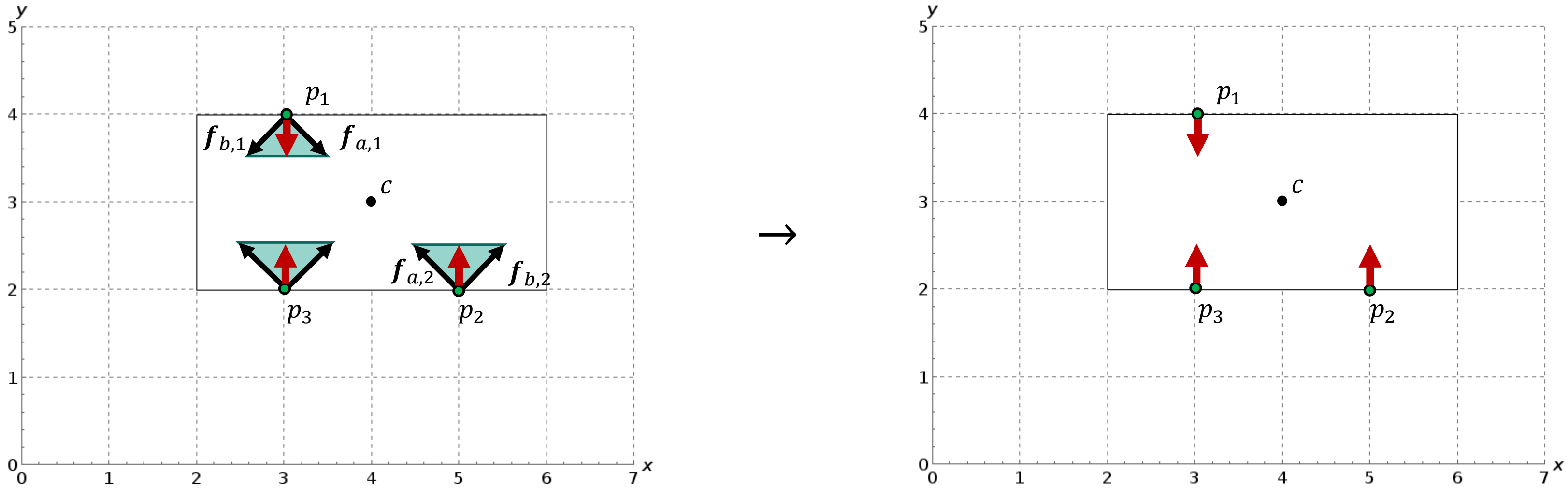
- What is $\text{pos}(G)$?

$$\text{pos}(G) = \left\{ \begin{pmatrix} f_x \\ f_y \\ \tau \end{pmatrix} \in \mathbb{R}^3 \right\} = \mathbb{R}^3$$



- The grasp is force-closed

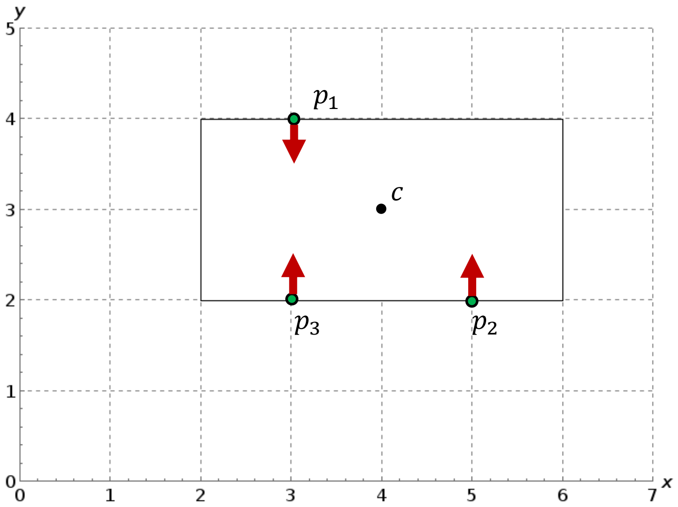
Exercise 3: Three-finger grasp, Bonus

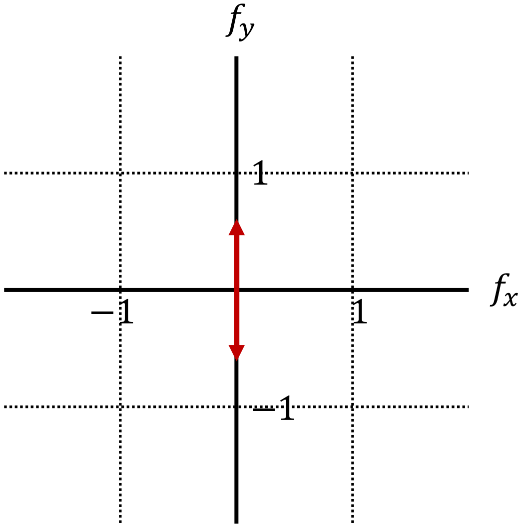


Is the three-finger grasp **form-closed**?

→ **Form-closure:** Assume point contact without friction

Exercise 3: Three-finger grasp, Bonus





Is the three-finger grasp **form-closed**?

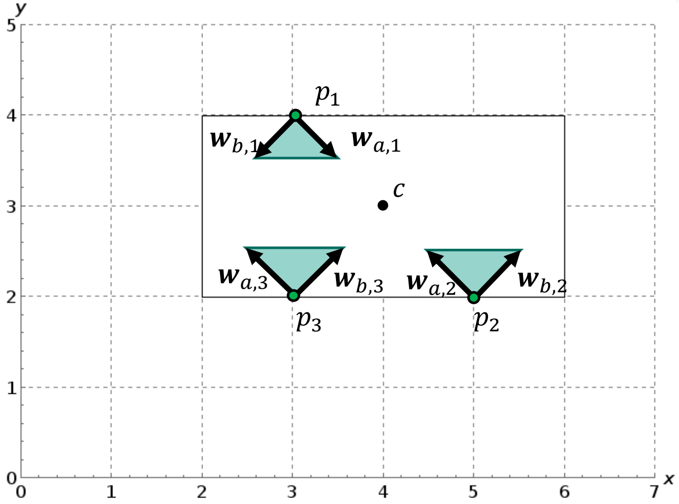
→ The grasp cannot counteract forces $f_x > 0$ or $f_x < 0$

Exercise 3: Force Closure

- Are the following grasps force-closed?

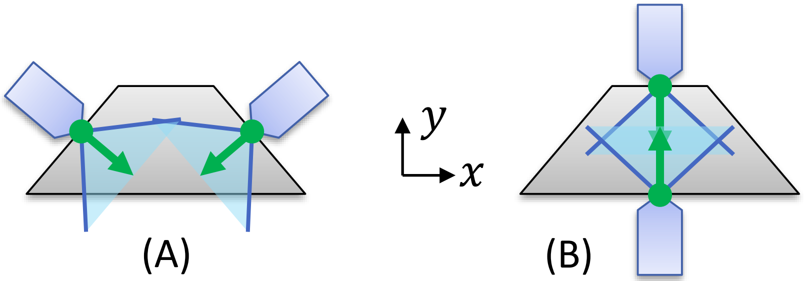
   1. Two-finger grasp: p_1, p_2

   2. Three-finger grasp: p_1, p_2 and p_3



- How would you calculate the **ε-metric** for the two grasps?

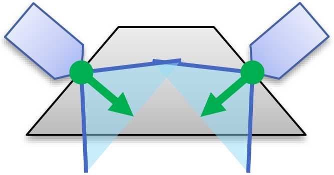
Grasp Quality: ε-Metric



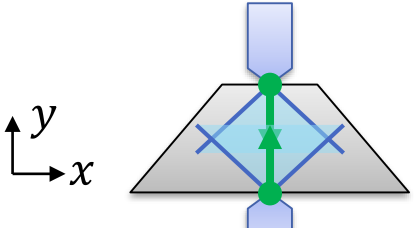
- Observe Grasps (A) and (B):
- Which grasp is **force-closed**?
  - (A) and (B)
- Are both grasps **equally good**?
  - With (A), high normal forces have to be excerted to generate friction forces in y-direction.
  - With (B), it is simpler to generate forces in all directions.
- How can this be **quantified**?

Grasp Quality: ε-Metric

- Observe Grasps (A) and (B):



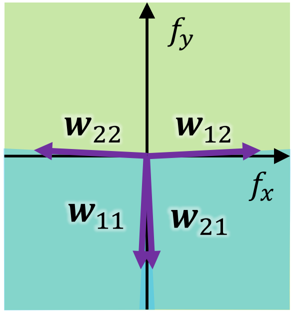
(A)

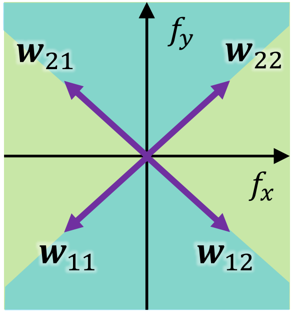


(B)

- Which grasp is **force-closed**?
  - (A) and (B)

Compare the f_x, f_y-planes of the wrench spaces:



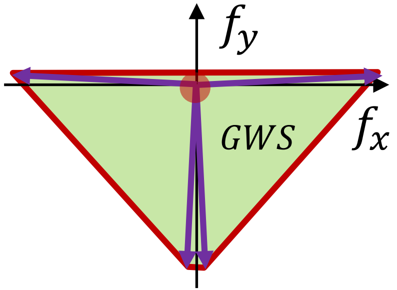


Grasp Quality: Grasp-Wrench-Space

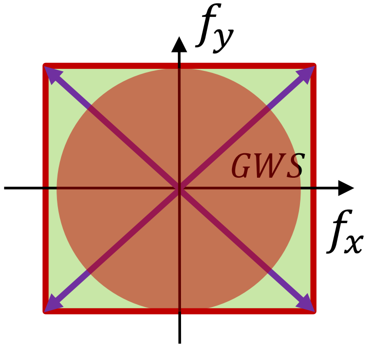
The **Grasp-Wrench-Space** GWS is the convex hull of the $\boldsymbol{w}_i$

$$GWS = \text{conv}(\{\boldsymbol{w}_i\}) = \{\sum_{i=1}^{m} k_i \boldsymbol{w}_i \mid k_i \geq 0 \text{ and } \sum_{i=1}^{m} k_i = 1\}$$

How could one define a **measure for the quality** of a grasp using GWS?



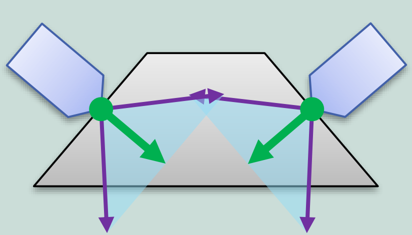
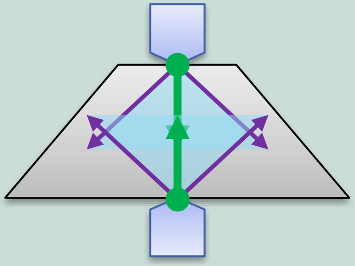
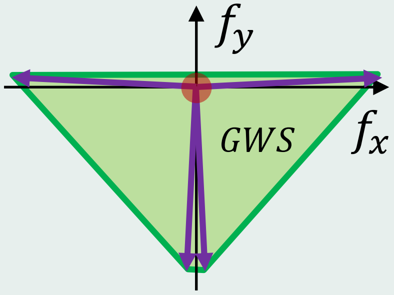
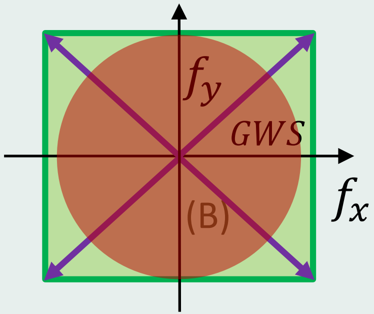
> The **ε-metric** is the **radius of the largest sphere** around the origin of the GWS that is still completely contained in GWS.
>
> It is sometimes also called the Grasp-Wrench-Space metric.



Intuition:

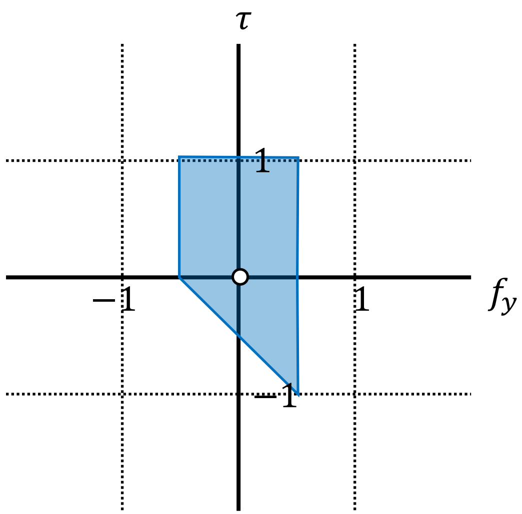
- ε is the strenght of the smallest wrench which brakes the grasp.
- The grasp withstands all wrenches with a strength of less than ε.
- If $\varepsilon > 0$, the grasp is force-closed.
- The larger ε, the more „stable“ the grasp.

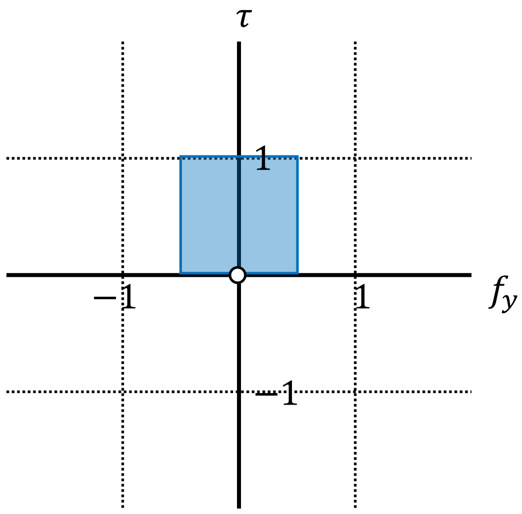
ε-Metric: Examples

Grasp	(A)	(B)
2D workspace		
f_x, f_y-Plane of the Grasp-Wrench-Space with ε-sphere around the origin		
ε_{xy}-metric in f_x, f_y-plane	$\varepsilon_{xy} = 0.0707$	$\varepsilon_{xy} = 1$

Exercise 3.3: ε-Metric

Three-finger grasp

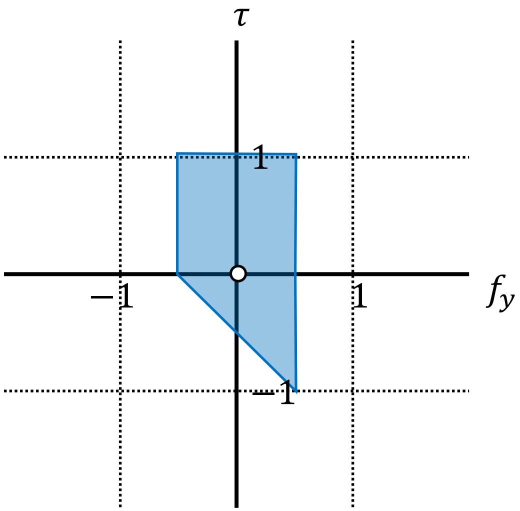


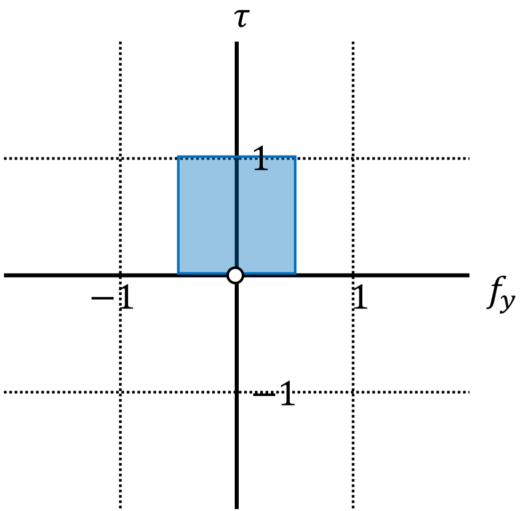


Two-finger grasp

Exercise 3.3: ε-Metric

Three-finger grasp



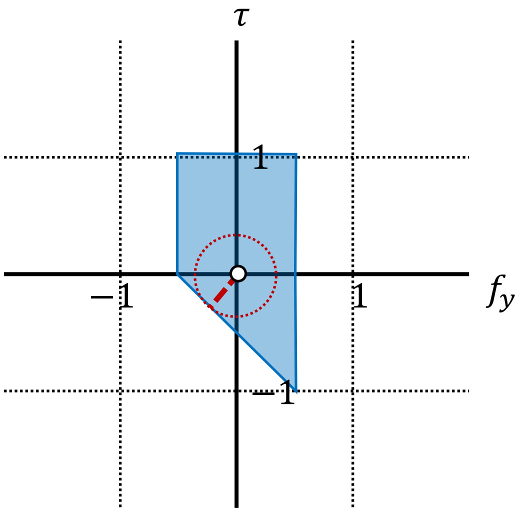


Two-finger grasp

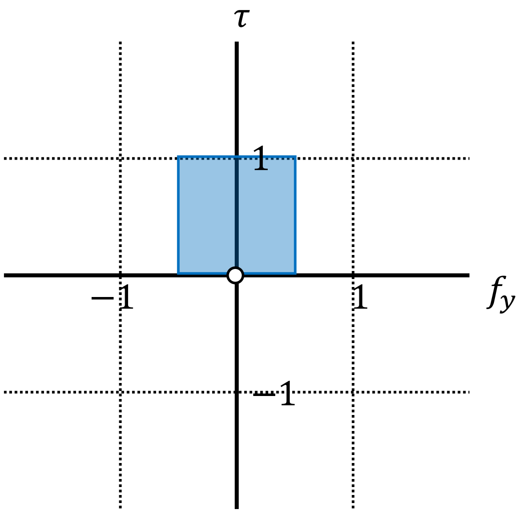
1. Calculate wrenches at the contact points
2. Draw the Grasp Wrench Space (convex hull of the wrenches)
3. Determine the minum distance from the origin to the edge of the Grasp Wrench Space

Exercise 3.3: ε-Metric

Three-finger grasp



$\varepsilon > 0$



$\varepsilon = 0$

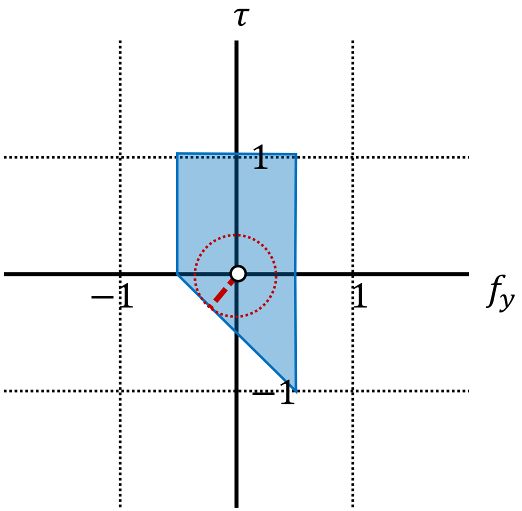
Two-finger grasp

1. Calculate wrenches at the contact points
2. Draw the Grasp Wrench Space (convex hull of the wrenches)
3. Determine the minum distance from the origin to the edge of the Grasp Wrench Space

Exercise 3.3: ε-Metric, Bonus

What is the value range of the $\boldsymbol{\varepsilon}$-metric?

a) $\varepsilon \in (-\infty, \infty)$
b) $|\varepsilon| \ll 1$
c) $\varepsilon \in (0, \infty)$
d) $\varepsilon \in [0, \infty)$
e) $\varepsilon \in [0, 1]$



Exercise 3.3: ε-Metric, Bonus

What is the value range of the $\boldsymbol{\varepsilon}$-metric?

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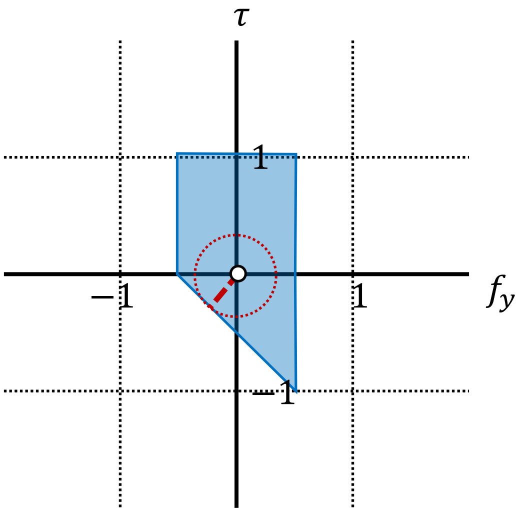
d) $\boldsymbol{\varepsilon \in [0, \infty)}$
e) $\varepsilon \in [0, 1]$

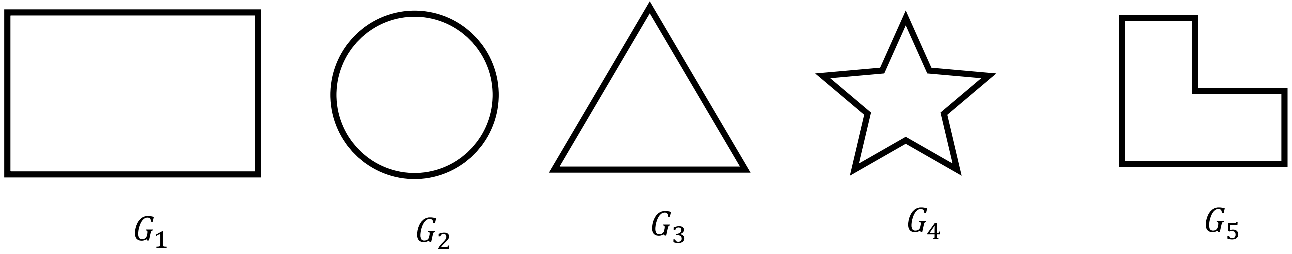


Exercise 4: Medial Axes

Karlsruher Institut für Technologie

- The **medial axis** of a two-dimensional region $G \subset \mathbb{R}^2$ is the set of centers of the **maximum circles** in G.
- A circle K is a maximum circle in G if there is no circle K' for which $K \subset K' \subseteq G$ is true:
  - $\mathrm{K} \subseteq G$ and
  - $\neg \exists K' : K \subset K' \subseteq G$

- Draw the medial axes of the regions $G_1, \ldots, G_5$.

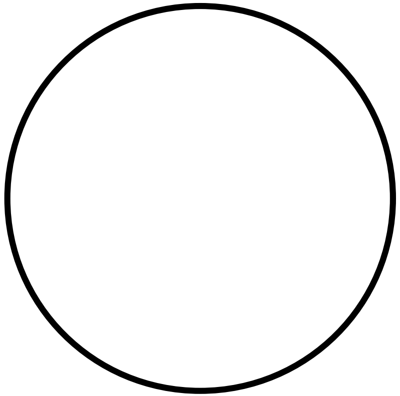


Grasp Planning with Medial Axes

- The medial axis (Blum 1967) describes the **topological skeleton** of the object
- In 3D: Consider center of spheres instead of center of circles
- Grasp candidates can be generated using heuristics
  - High percentage of stable and “natural” grasps
- Advantages:
  - Good approximation of the object geometry
  - Details are retained
  - Good description of symmetries

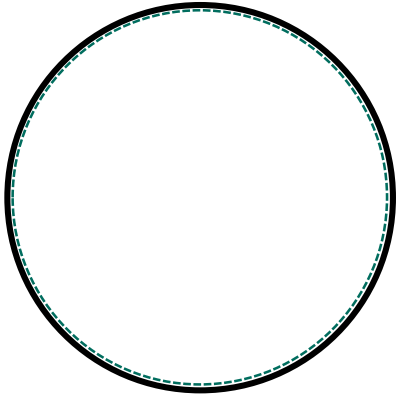
H. Blum, Models for the Perception of Speech and Visual Form. A transformation for extracting new descriptors of shape, Cambridge, Massachusetts: MIT Press, 1967, pp. 362–380.

Exercise 4: Medial Axes



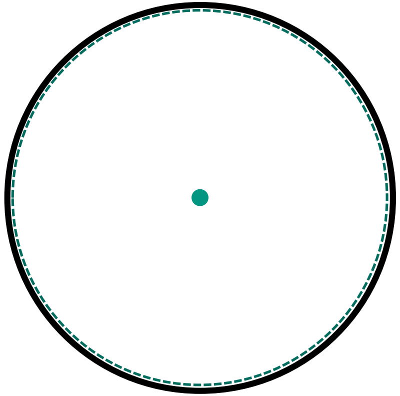
G_2

Exercise 4: Medial Axes



G_2

Exercise 4: Medial Axes

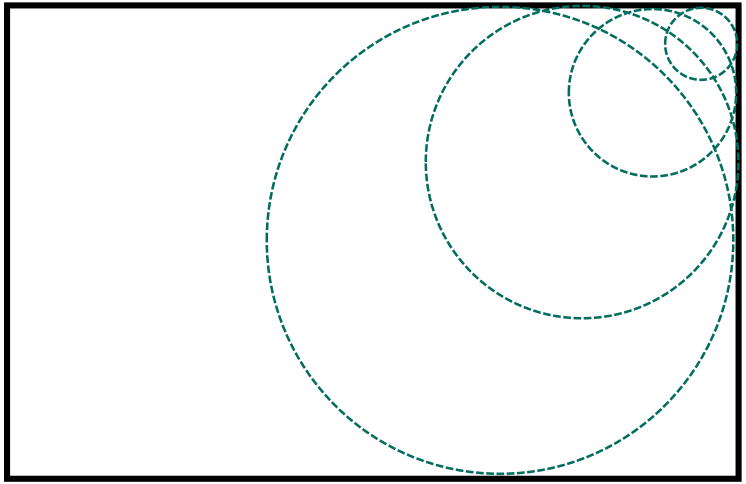


G_2

Exercise 4: Medial Axes

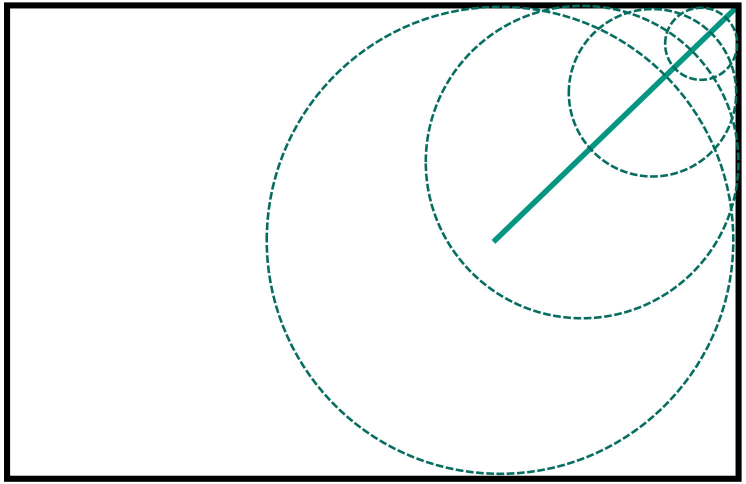
G_1

Exercise 4: Medial Axes



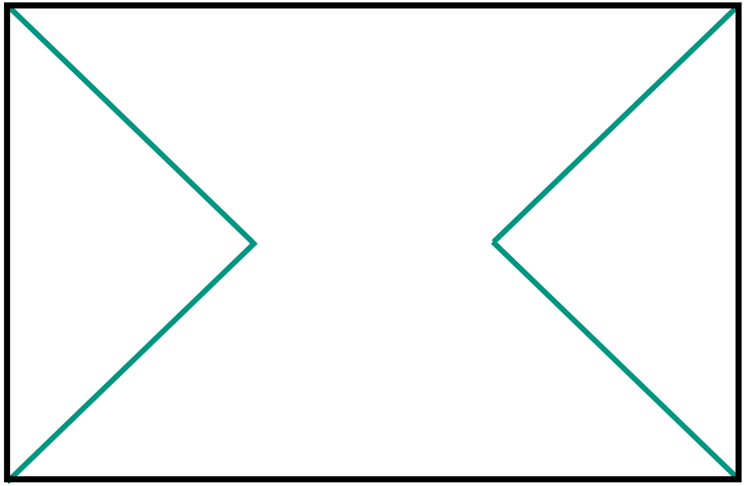
G_1

Exercise 4: Medial Axes



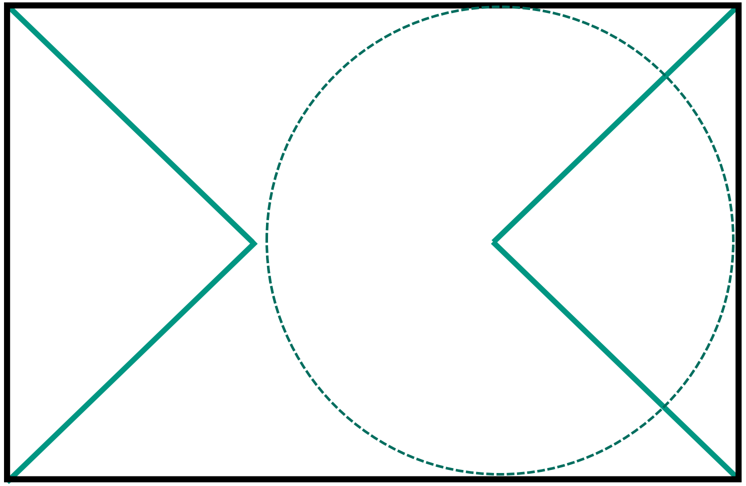
G_1

Exercise 4: Medial Axes



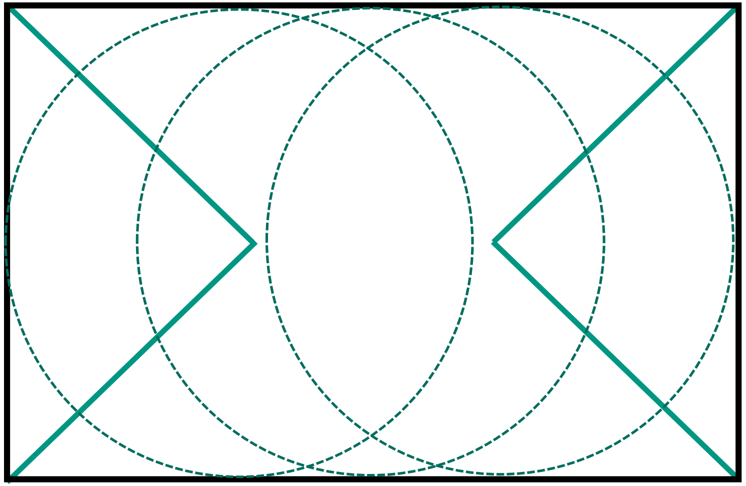
G_1

Exercise 4: Medial Axes



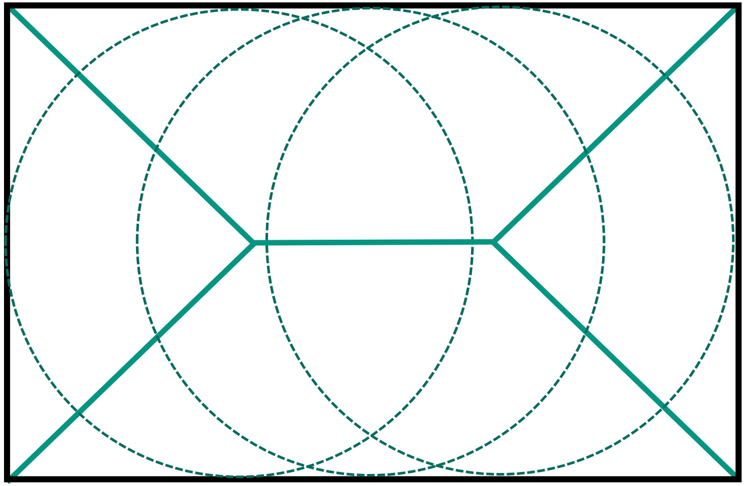
G_1

Exercise 4: Medial Axes



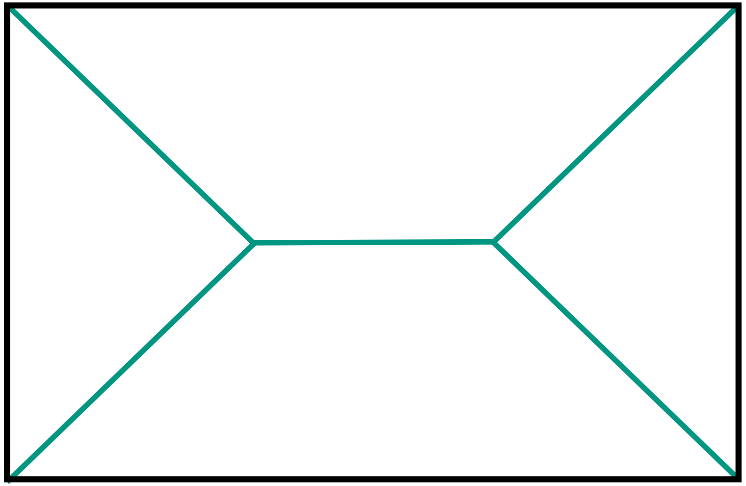
G_1

Exercise 4: Medial Axes



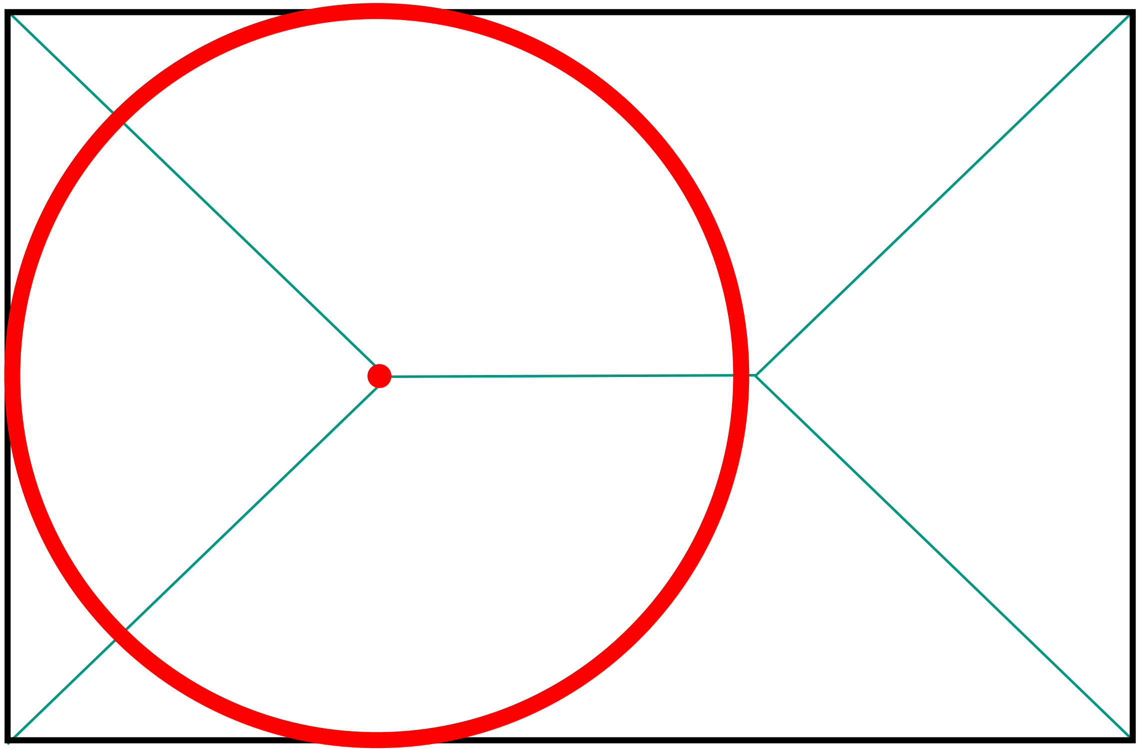
G_1

Exercise 4: Medial Axes

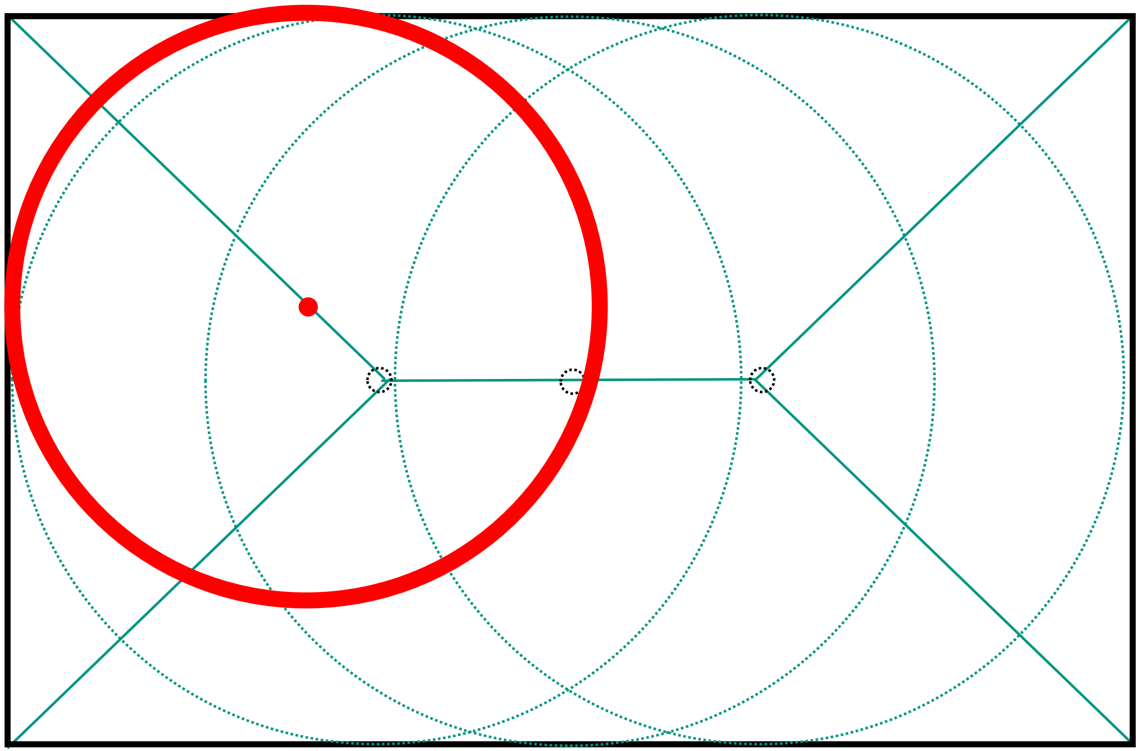


G_1

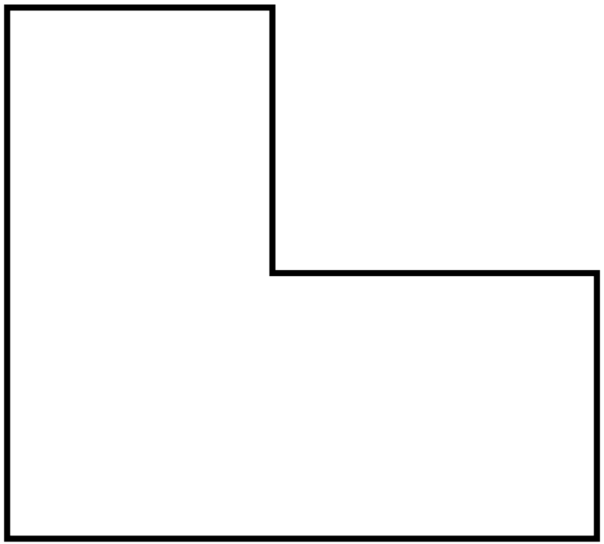
Exercise 4: Medial Axes



Exercise 4: Medial Axes

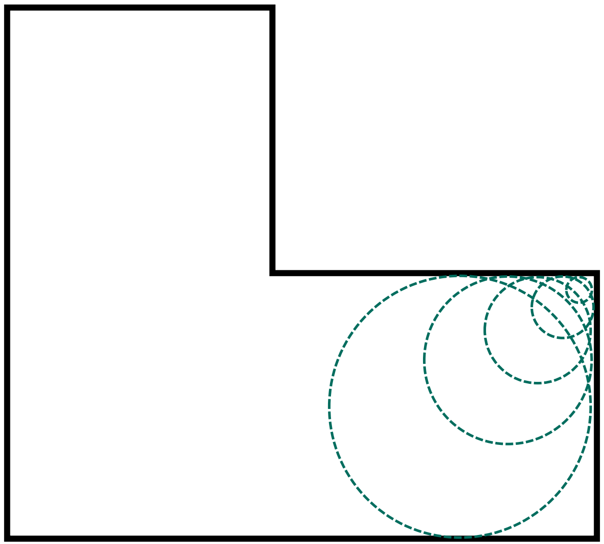


Exercise 4: Medial Axes



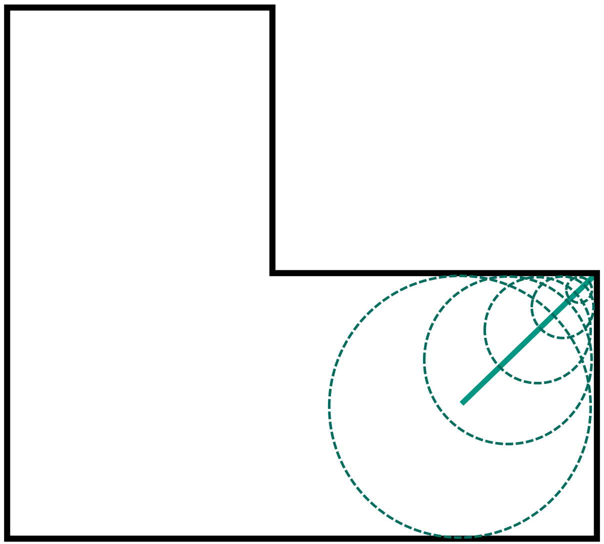
G_5

Exercise 4: Medial Axes



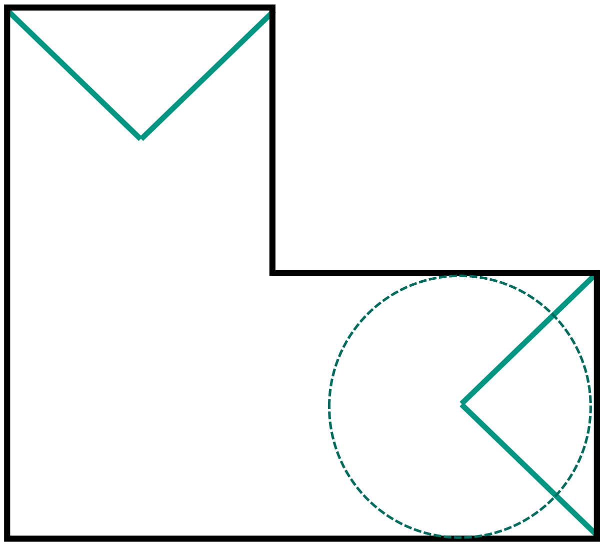
G_5

Exercise 4: Medial Axes



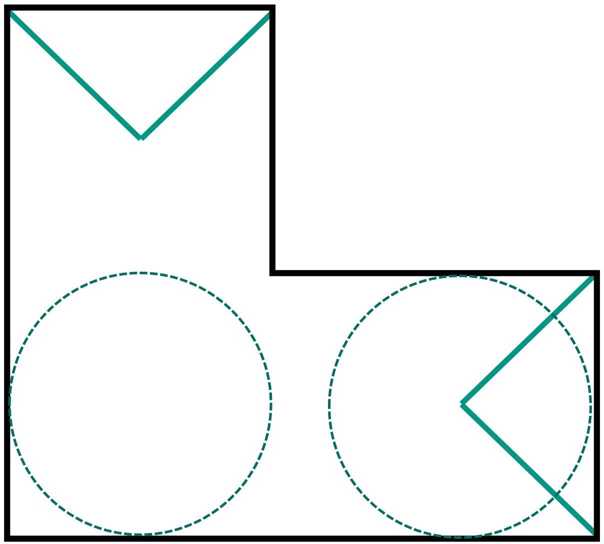
G_5

Exercise 4: Medial Axes



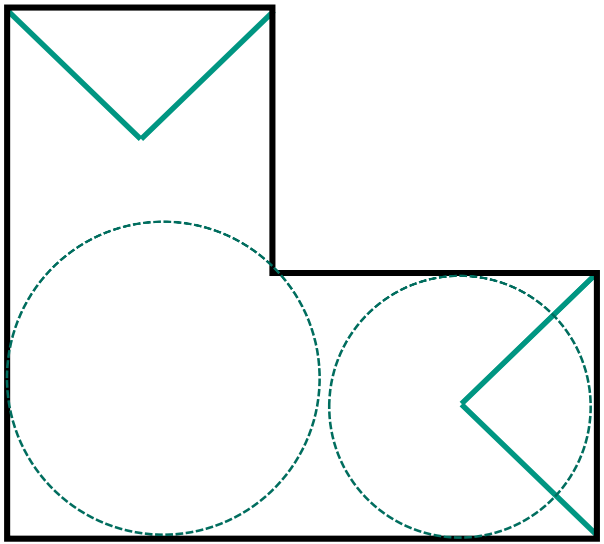
G_5

Exercise 4: Medial Axes



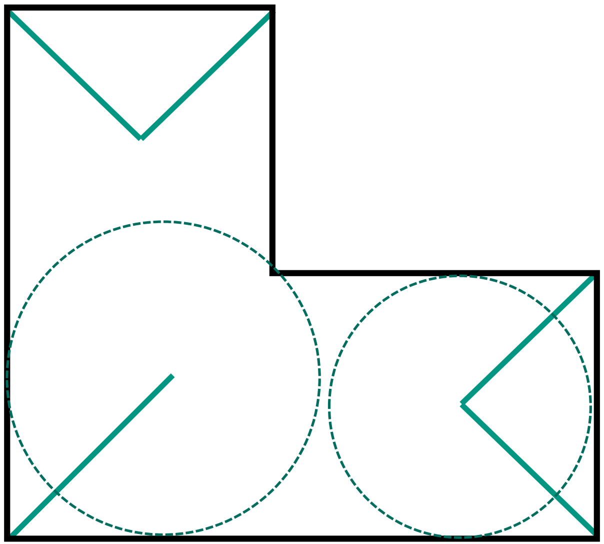
G_5

Exercise 4: Medial Axes



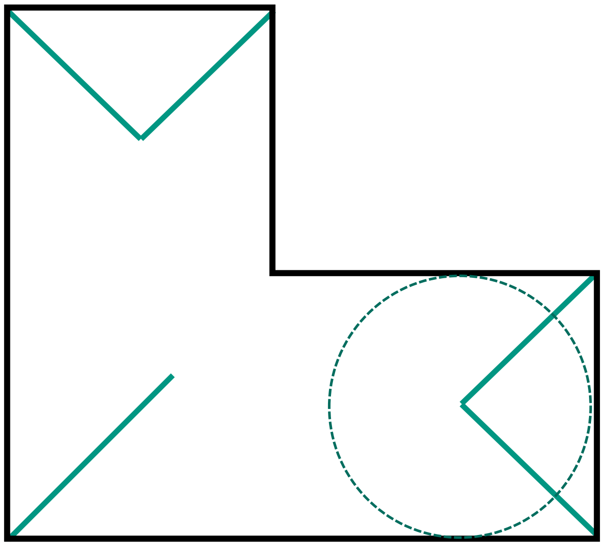
G_5

Exercise 4: Medial Axes



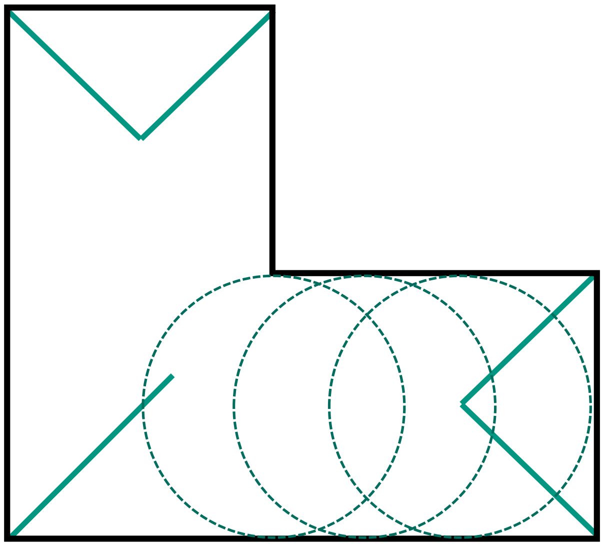
G_5

Exercise 4: Medial Axes



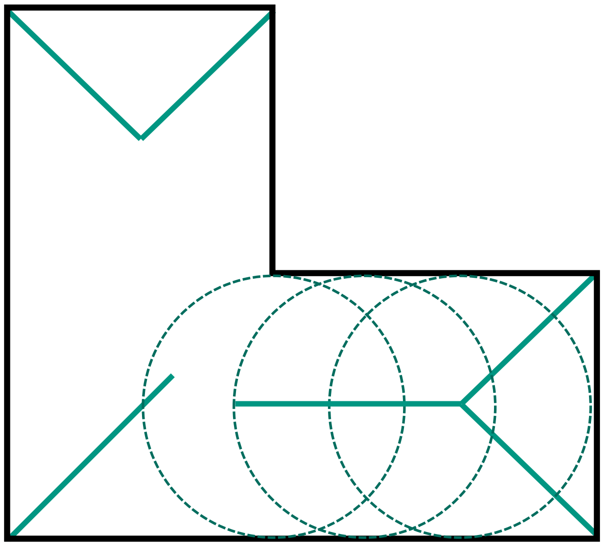
G_5

Exercise 4: Medial Axes



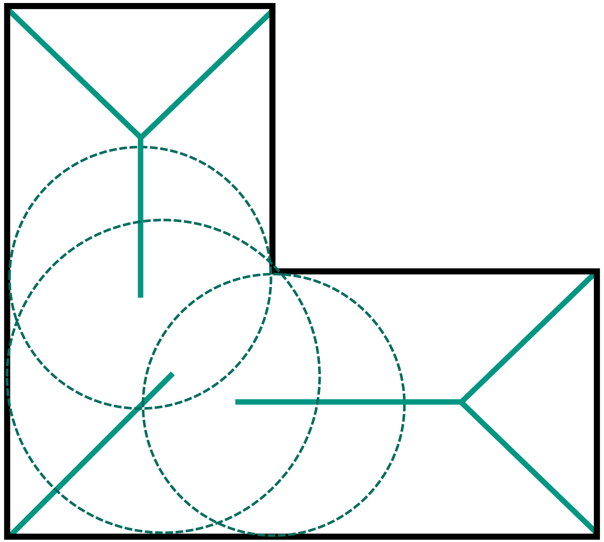
G_5

Exercise 4: Medial Axes



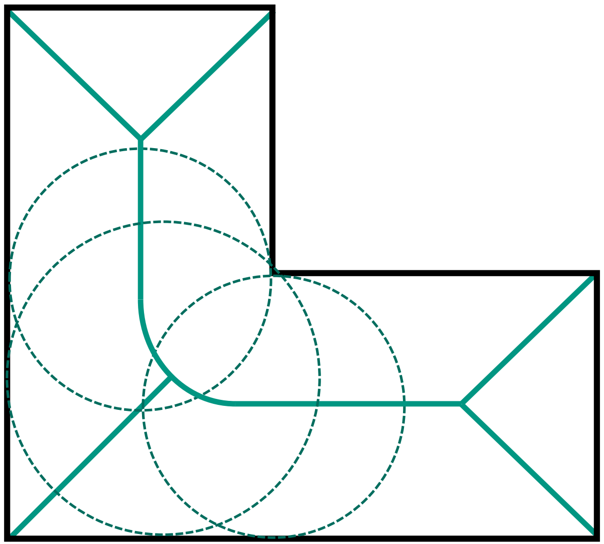
G_5

Exercise 4: Medial Axes



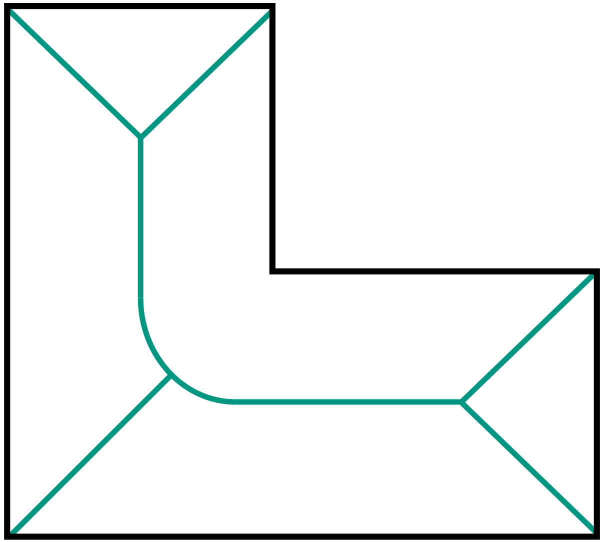
G_5

Exercise 4: Medial Axes



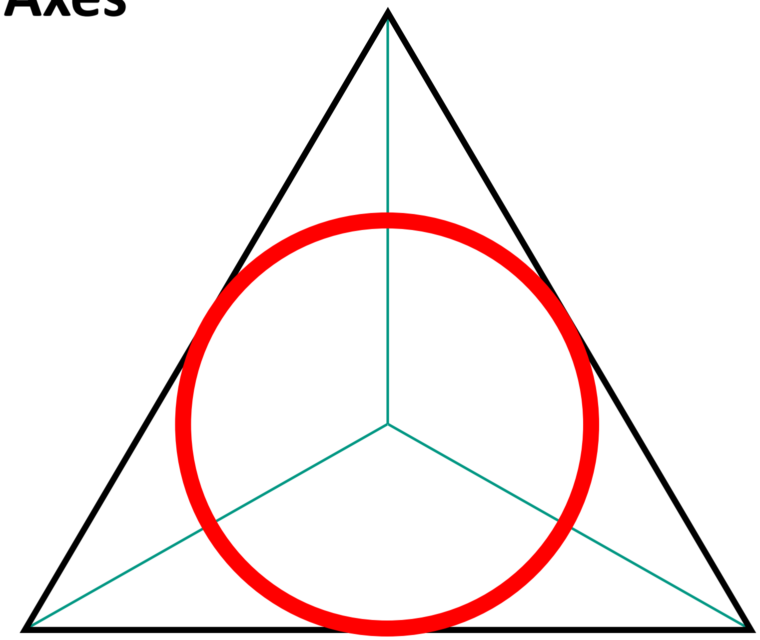
G_5

Exercise 4: Medial Axes



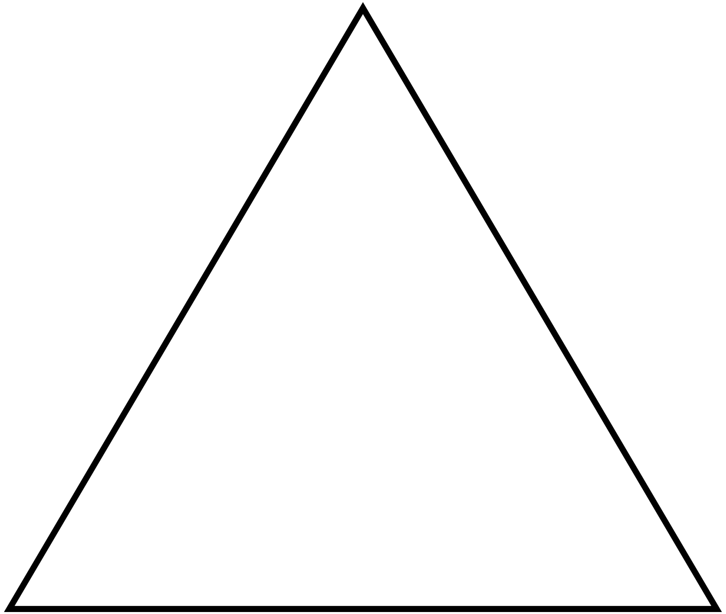
G_5

Exercise 4: Medial Axes



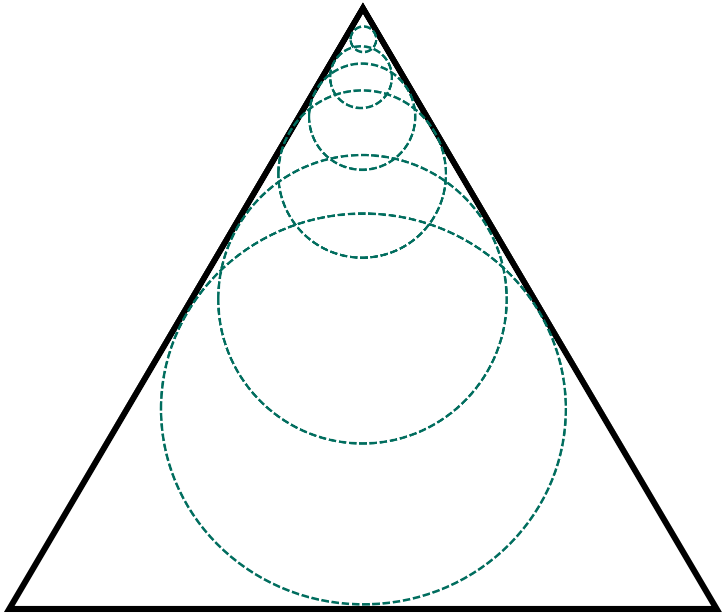
G_3

Exercise 4: Medial Axes



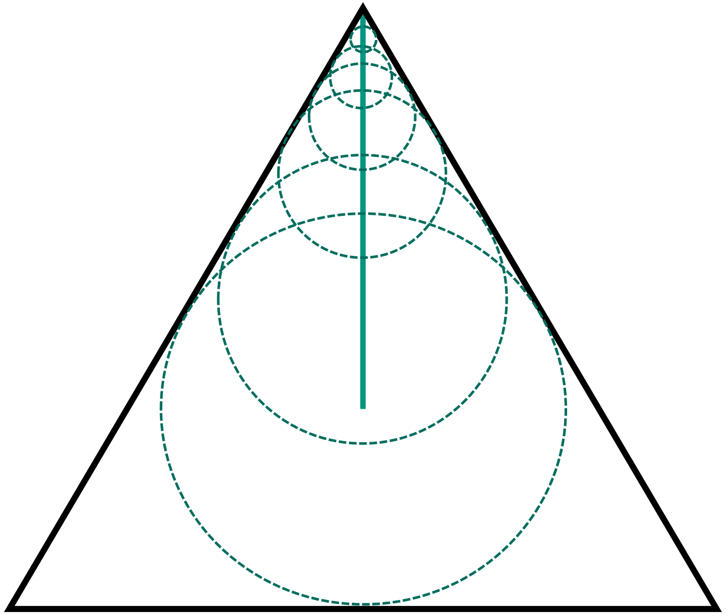
G_3

Exercise 4: Medial Axes



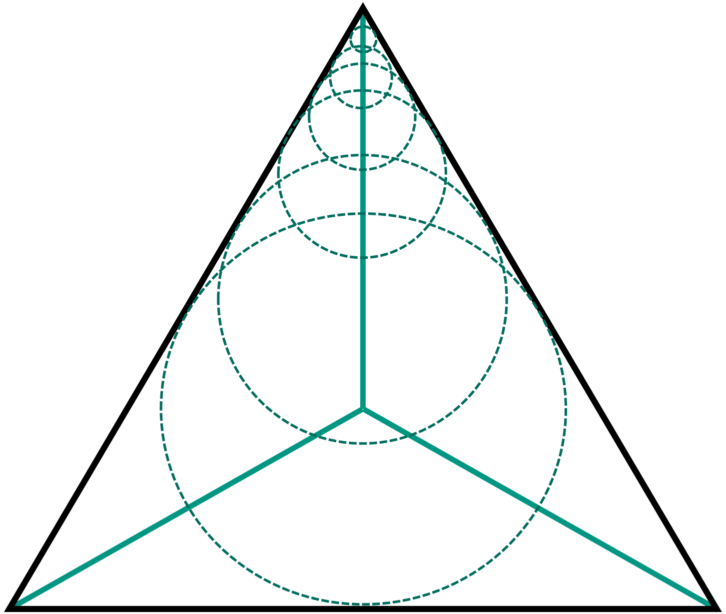
G_3

Exercise 4: Medial Axes



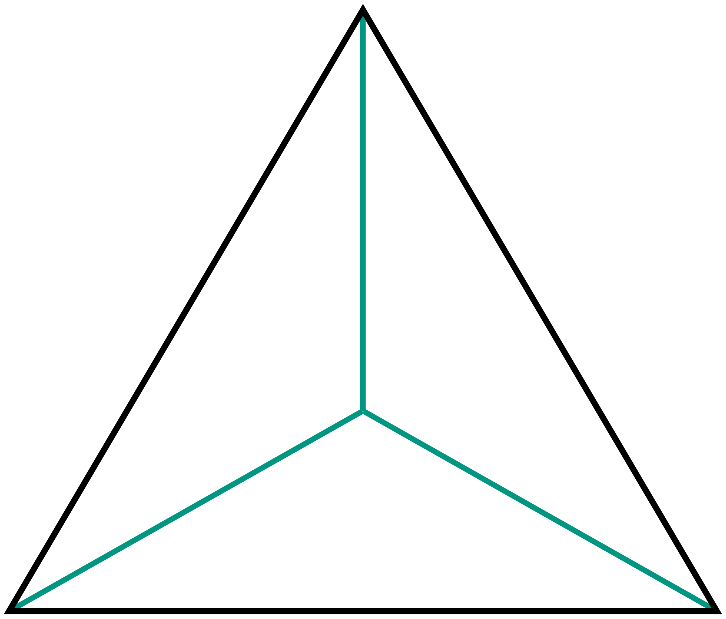
G_3

Exercise 4: Medial Axes



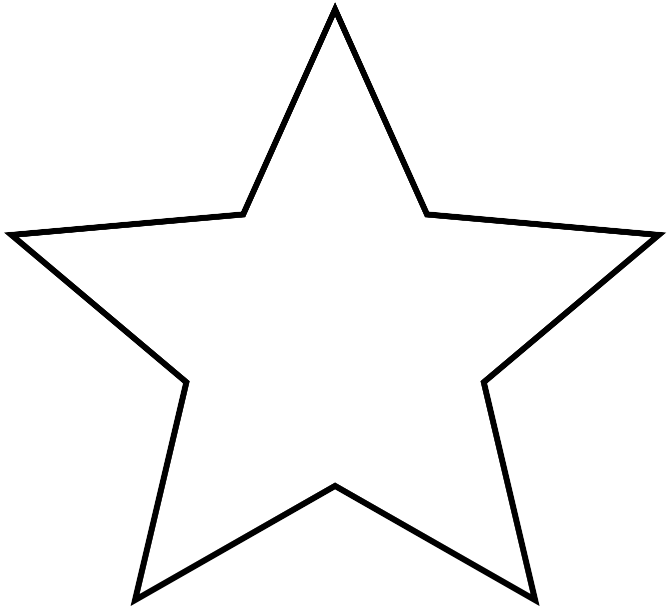
G_3

Exercise 4: Medial Axes

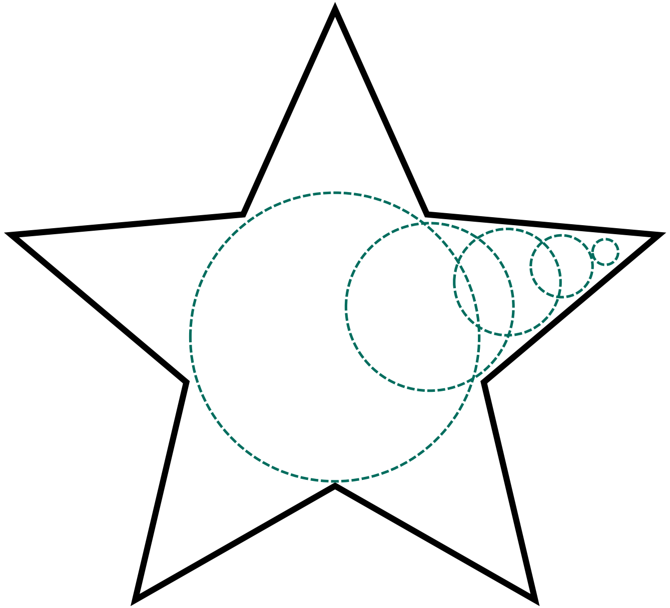


G_3

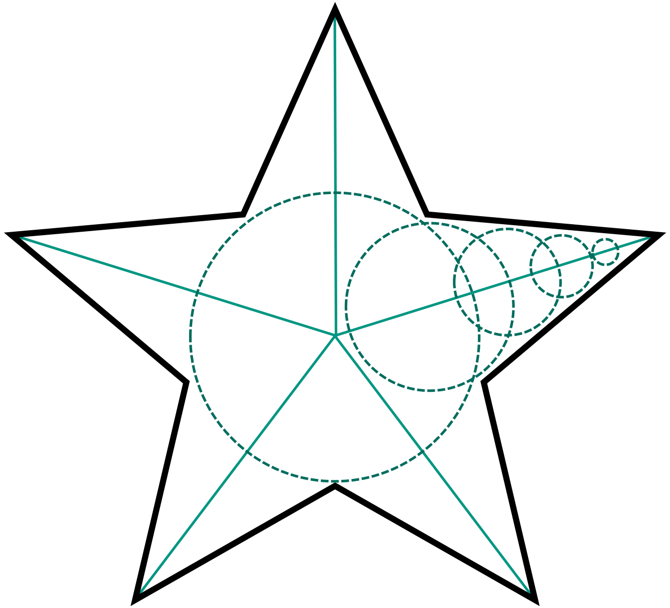
Exercise 4: Medial Axes



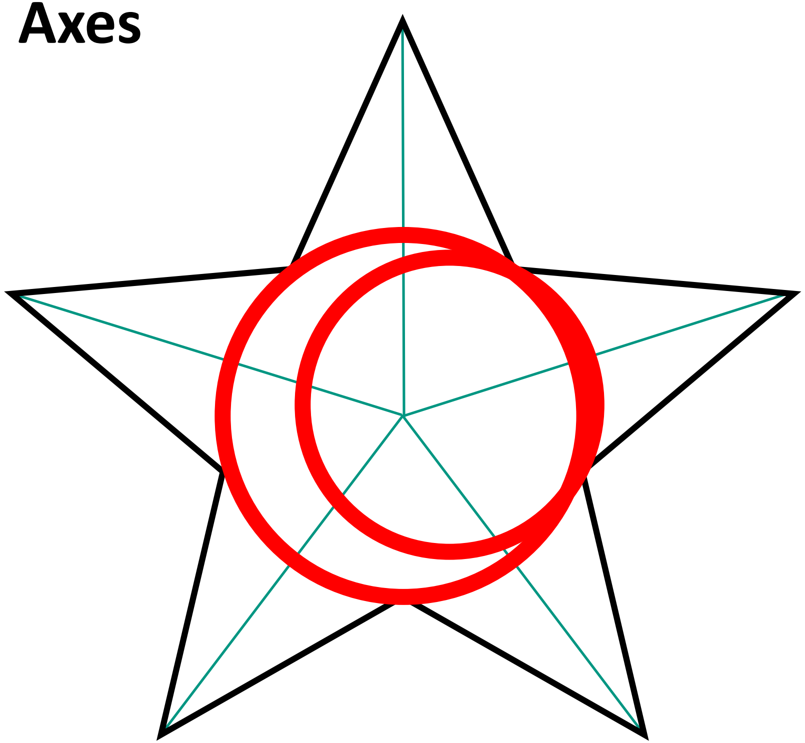
Exercise 4: Medial Axes



Exercise 4: Medial Axes



Exercise 4: Medial Axes



G_4

Exercise 4: Medial Axes